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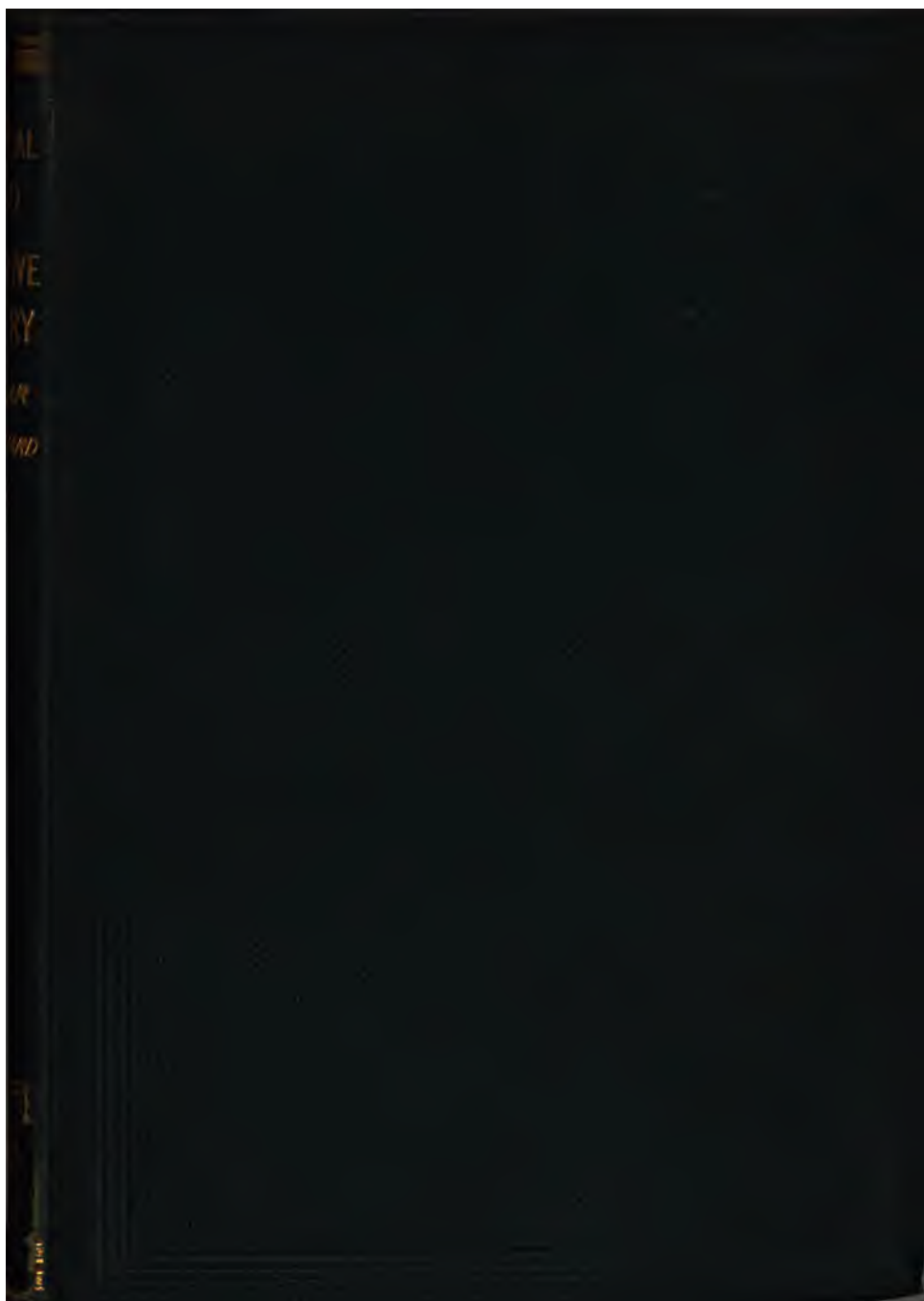
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NOTE-BOOK
ON
PRACTICAL SOLID
OR
DESCRIPTIVE GEOMETRY

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NOTE-BOOK
ON
PRACTICAL SOLID
OR
DESCRIPTIVE GEOMETRY
CONTAINING
PROBLEMS WITH HELP FOR SOLUTIONS.

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183. g. 61.



PREFACE.

IN teaching a large class, if the method of lecturing and demonstrating from the black-board only is pursued, the more intelligent students have generally to be kept back, from the necessity of frequent repetition, for the sake of the less promising ; if the plan of setting problems to each pupil is adopted, the teacher finds a difficulty in giving to each sufficient attention. A judicious combination of both methods is doubtless the best, though this is not always easy of attainment in working with numbers—the use of this book may help in accomplishing it.

It is suggested that at the beginning of a chapter, and in some cases with each problem, the teacher should give a black-board explanation, carefully pointing out any fresh steps, before sending his pupils to their work. The number of examples in each chapter to be worked out by the student is, of course, left to the teacher's judgment of the progress and requirements of his pupil.

The student will here be assisted, it is believed, because he will find help given when necessary, and thus will often be able to go on with his work by himself, with the satisfaction also of feeling that he is progressing systematically. Moreover he will be-

come practised in dealing with written questions—a point not to be lost sight of by those who are preparing for examinations.

The Second Edition has been enlarged by the addition of chapters on the straight line and plane with explanatory diagrams and exercises, on tangent-planes, and on the cases of the spherical triangle.

It is hoped that the work, thus rendered more complete, may prove still more useful as a class-book and means of self-instruction to the various and constantly increasing classes of students for whom it is designed. It was originally intended as an aid in teaching the Mechanical Drawing Class at the Royal School of Mines from Professor Bradley's *Elements of Practical Geometry*. The authors of this work were associated with him in his duties at King's College, London, and the Royal Military Academy, and learnt practically the value of his treatise; but the cost of that work has rendered it inaccessible to many for whom the present book may be available.

A greater number of diagrams have not been added, in order that students may be thrown upon their own resources, and encouraged to consider the *principles* upon which their work proceeds, more than they would probably do if there were figures always at hand for reference.

October, 1871.

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ELEMENTARY SOLID OR DESCRIPTIVE GEOMETRY.

ELEMENTARY EXPLANATIONS, DEFINITIONS AND THEOREMS.

“The object of Descriptive Geometry is the invention of methods by which we may represent upon a plane having only two dimensions, namely length and breadth, the form and position of a body which possesses three dimensions, namely length, breadth, and height.”

HALL's *Elements of Descriptive Geometry*.

IN Descriptive Geometry Solids are represented by their drawings or *projections* on two planes conceived at right angles to one another, intersecting in a line called the *ground line xy*, and named from their usual positions the *horizontal* and *vertical planes of projection*. See Fig. 1.

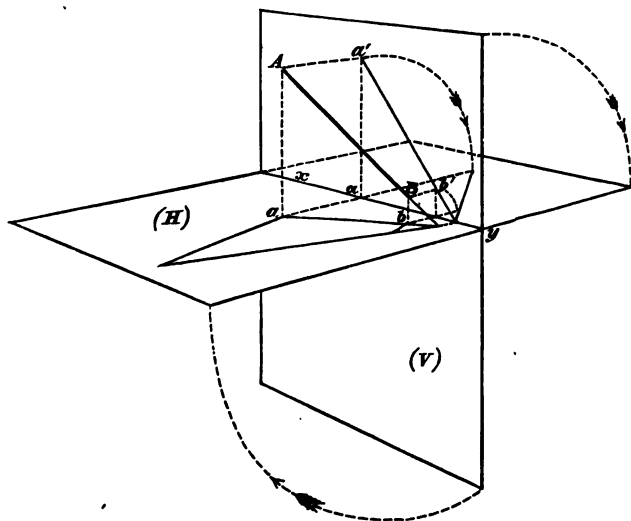
These planes form four dihedral angles.

Drawings or *projections* on the horizontal plane are called *plans*; on the vertical plane *elevations*.

The *plan* or *horizontal projection* of any point *A* in space is the foot of the perpendicular let fall from point *A* on the horizontal plane, and is marked *a*; the *elevation* of *A* is marked *a'* and is the point of intersection of the perpendicular from *A* on the vertical plane. See Fig. 1.

The perpendicular Aa from the given point A , upon the given plane is termed the *projector* or *projecting-line* of that point, and the plane its *plane of projection*. Plane (H) Fig. 1.

Fig. 1.



The plane $ABab$ containing the given straight line AB , and perpendicular to the plane of projection, is called the *projecting plane* of that line; and the intersection ab of these two planes the *projection* of the line, i.e. its *plan*, or in the case of AB and $a'b'$ its *elevation*.

The projection of a line may also be defined to be the sum of the projections of its points.

If the given line be a curve, not lying in a single plane, the Surfacies containing all its projectors for a given plane of projection, is termed the *projecting surface* of the curve :

the intersection of this surface and plane is the *projection* of the curved line.

Hence it will appear that a line is defined or expressed for the purpose of 'Descriptive Geometry,' by the aid of two planes of projection and two *projecting planes* or *projecting surfaces*.

Solid bodies, as they possess three dimensions, length, breadth, and height, require to be represented or projected upon both planes of projection, so that 'height' as well as 'length' and 'breadth' may be exhibited or determined by the drawings.

But as we can have for drawing only one plane, that is, one flat sheet of paper, not two at right angles to each other, the upper part of the vertical plane is supposed to rotate backwards upon xy , its intersection with the horizontal plane, as upon a hinge, until it coincides and forms one plane with the horizontal. The plan a and elevation a' of any point A will then lie in the same perpendicular to the ground line.

This perpendicular will thus form a *locus* of points a and a' , that is to say, must contain points a and a' .

The distance of a from xy shows the distance of the point A from the vertical plane.

The distance of a' from xy shows the distance of the point A from the horizontal plane, i. e. its height.

In all constructions reference is continually made to these relations which may be thus stated for reference.

THEOREM I. The two projections of a point are in the same perpendicular to xy after the vertical plane has been turned into the horizontal.

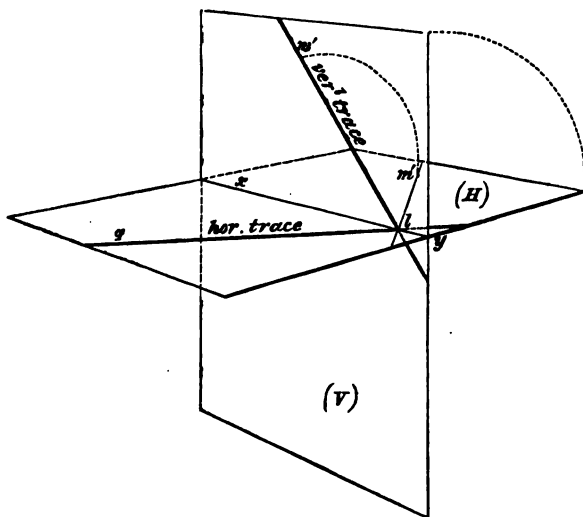
THEOREM II. The perpendiculars on xy from the projections of a point are respectively equal to the distances of the point from the planes;

i.e. The perpendicular $a'a$ from the vertical projection a' is equal to the height of the point.

The perpendicular aa from its *plan* is equal to the distance of the point from the vertical plane of projection.

The point in which any line or line produced pierces the horizontal plane is called its *horizontal trace*: that in which it pierces the vertical plane its *vertical trace*.

Fig. 2.



The line in which a plane or surface cuts the horizontal plane is called its *horizontal trace*; that in which it cuts the vertical plane its *vertical trace*. Fig. 2. These two traces

intersect in the ground line xy , unless by the position of the plane they are parallel to it; or the plane is parallel to one of the planes of projection, in which case there will be only one trace.

Notation.

A point in space is indicated by a capital letter A , its projections by a and a' ; the accented letter denoting the vertical projection or elevation of the point.

Similarly a line in space is denoted by the capital letters AB and its projections by ab , $a'b'$.

A plane is denoted by three letters hav' , one on each trace and the third on their point of intersection in xy . Figs. 5 and 2.

If two planes be perpendicular to each other, every line drawn from a point in one plane perpendicular to the other lies wholly in the former; and therefore meets the latter in the common section of the planes.

From this and the definitions it follows; that

THEOREM III. (1) A point in one plane of projection has its projection on the other *in* xy ;

i.e. If the point be *in* the vertical plane, its *plan* is *in* xy . If in the horizontal plane, its *elevation* is *in* that line.

(2) If a given plane be perpendicular to one plane of projection, its trace on that plane contains the projections on the same of all points, lines, &c. in the given plane.

(a) Thus if an inclined plane be perpendicular to the vertical plane, the elevations of points in the inclined plane

will be in its *vertical* trace. Ex. gr. A circle of 3 inches diameter in a plane 30° which is perpendicular to the vertical plane, has a segment of 3 inches of the vertical trace for its *elevation*.

(β) A plane inclined to the vertical plane of projection and perpendicular to the horizontal plane, has the *plans* of all lines in it in its *horizontal* trace. Accordingly the plans of the vertical faces of a right prism resting on its base coincide with the lines forming the sides of that base. See Fig. 5.

THEOREM IV. The projections of parallel straight lines are parallels.

THEOREM V. The projections of a perpendicular to a plane are perpendiculars to the traces of the plane ;
i.e. The *plan* of the perpendicular is at right angles to the horizontal trace ; the *elevation* of the perpendicular is at right angles to the vertical trace.

THEOREM VI. If a *straight* line lies in a plane and has traces, they will be *in* those of the plane. Hence, if a plane is required which shall contain a given line, the traces of the line must be determined and those of the plane drawn through them.

THEOREM VII. The angle which the tangent plane to a right cone makes with the base of the cone is measured by the plane angle which a generatrix of the cone makes with the base.

THEOREM VIII. If a plane touches a conic surface and a second plane intersects them, the *trace* of the tangent plane on the intersecting one is a tangent to the trace of the conic surface on the same.

THEOREM IX. (α) The orthographic projection, i.e. the plan or elevation, of a finite straight line is equal to the original line if the latter be parallel to the plane of projection. The projection diminishes as the angle of inclination of the line increases, and becomes a minimum, i.e. a point, when the angle of inclination reaches 90° : or when the line is perpendicular to the plane*.

(β) The projection of a rectilineal angle on a plane parallel to that of the angle is an angle equal to the given one.

The projection on the same plane, of the circular arc subtending that angle will also be equal to that arc. See Figs. 21 and 22.

Similarly, the projection of any figure on a plane parallel to it is a figure equal in all respects to the given one.

And the area of the *projection* of a given figure decreases as the dihedral angle of the plane of projection and the plane of the given figure increases.

When this angle is a right angle the projection of the figure is a minimum, i.e. a straight line as before indicated.

These considerations will determine the positions of the planes of projection selected, in reference to the data or quæsitæ of a given problem.

A particular operation called "*constructing*" a plane is often necessary which we now proceed to explain.

* The length of the projection of a finite line L inclined θ° to the plane of projection is $L \cos \theta$.

A plane is said to be *constructed* when it has been folded or revolved about its trace as an axis into a plane of projection : carrying with it all points lines and figures, which are contained in it.

The plane may thus revolve about its vertical or horizontal trace into the corresponding plane of projection.

This operation is necessary when it is required to show the real magnitude of an angle or figure lying in a single plane.

THEOREM X. Points so 'constructed' will lie or have their *loci* in perpendiculars to the trace used, drawn through their corresponding projections.

Thus if the *horizontal* trace be used the 'constructed' points will lie in perpendiculars to that trace through their respective *plans*, &c.

The reason for this relation or 'locus' will be obvious by considering that perpendiculars on the trace from points in the plane move in planes perpendicular to that axis.

Also if points have *fixed* positions with regard to the rotating plane, their projections upon it when in its second position will be in perpendiculars to the trace used, drawn through the elevations or plans of the points as that trace is the horizontal or vertical one,

These loci are to be made use of in determining the projection of a frustum of a solid on a plane which passes through it.

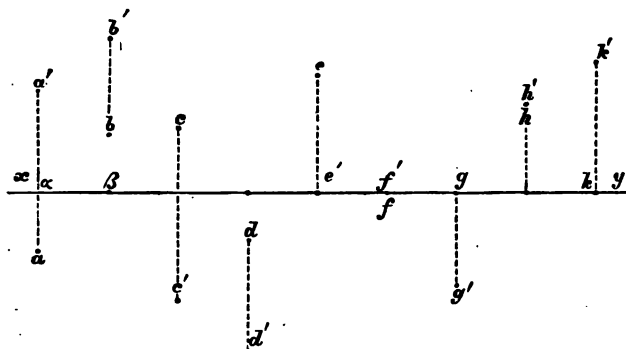
I.

Obs. To express points lines and planes see Problems 1 to 7.

DEFINITIONS ILLUSTRATED.

Fig. 3.

Points.



Examples of points determined in various positions by their projections.

Point *A* or *aa'* is in the 1st dihedral angle of the planes of projection.

„ *B* or *bb'* „ 2nd „ „ „ „ „

„ *C* or *cc'* „ 3rd „ „ „ „ „

„ *D* or *dd'* „ 4th „ „ „ „ „

„ *E* or *ee'* is in the horizontal and behind the vertical plane.

„ *F* or *ff'* is in both planes.

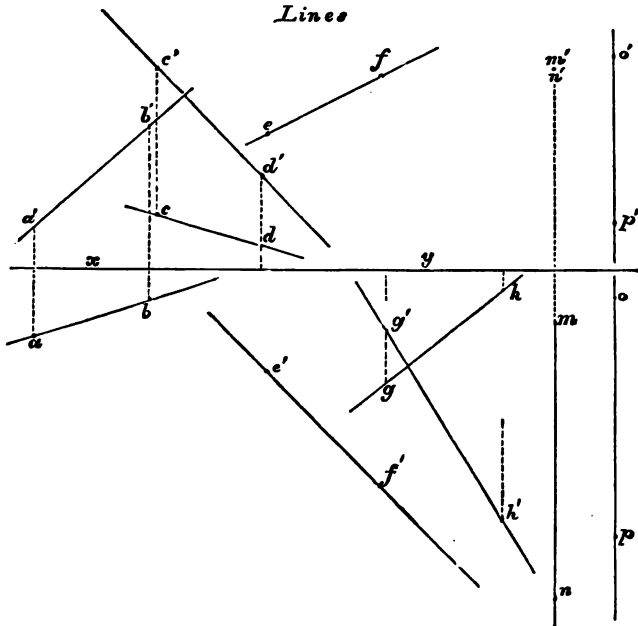
„ *G* or *gg'* is in the vertical and below the horizontal plane.

Point H or hh' is equidistant from the horizontal and vertical planes.

„ K or kk' is in the vertical and above the horizontal plane.

Observe. That to find the projections of points satisfying given conditions of position with regard to the planes of projection the relations indicated in Theorems I. and II. will suffice.

Fig. 4.
Lines

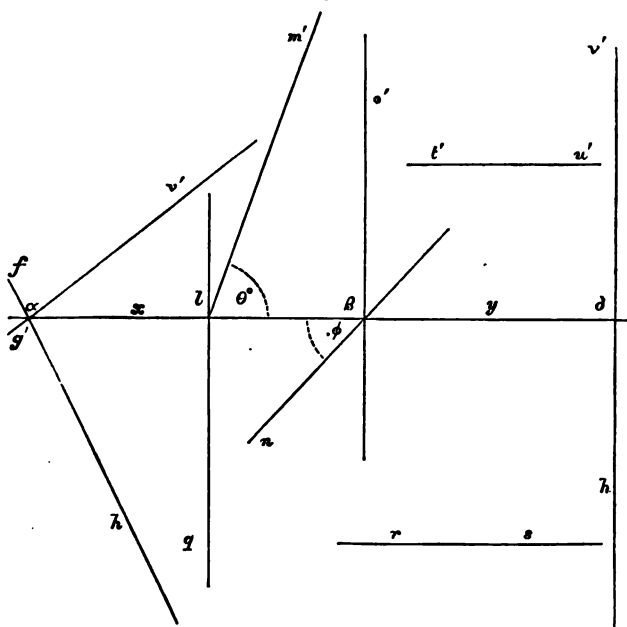


Examples of lines in various positions determined by their projections.

”	<i>CD</i> or <i>cd</i> , <i>c'd'</i>	”	2nd	”	”
”	<i>EF</i> or <i>ef</i> , <i>e'f'</i>	”	3rd	”	”
”	<i>GH</i> or <i>gh</i> , <i>g'h'</i>	”	4th	”	”
”	<i>MN</i> or <i>mn</i> , <i>m'n'</i>	”	1st	”	”

„ OP or op , $o'p'$ is in the 1st dihedral angle.

Planes



Examples of planes defined or expressed by their traces.

The portion included between the infinite branches ah and av' of the given plane is in the first dihedral angle of the planes of projection.

The portion between the branches av' and af is in the second dihedral angle.

The portion between af and ag' is in the third angle, and the portion between branches ag' and ah is in the fourth angle.

The plane glm' is perpendicular to the vertical plane and makes an angle of θ° with the horizontal.

The plane $n\beta o$ is at right angles to the horizontal and makes an angle of ϕ° with the vertical plane.

The plane $rs, t'u'$ is parallel to xy and meets both planes.

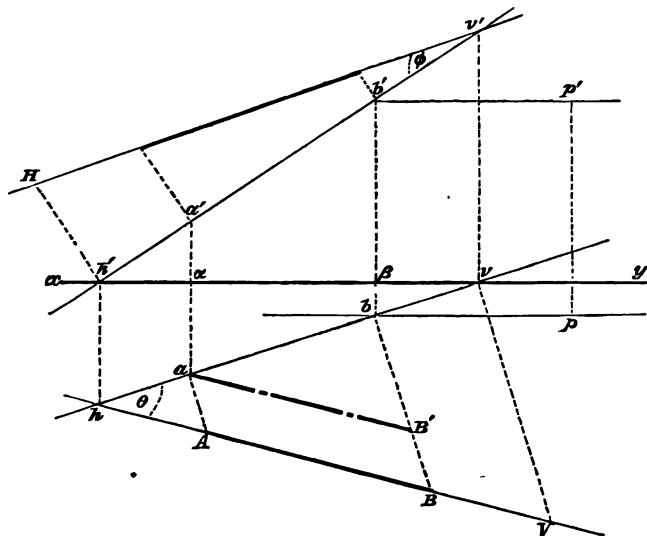
The plane $h\delta v'$ is at right angles to xy and therefore to both planes of projection.

PROBLEM I.

Given the projections $ab, a'b'$ of a finite straight line, to determine:

1. *The length of the segment AB of the given line, or the distance between points A and B.*
2. *The angle of inclination of the line to each plane of projection.*
3. *The traces of the given line and the distance between them.*

Fig. 6.



1. The line AB lies in each of its projecting planes: 'construct' or turn either of these into the corresponding plane of projection and the segment AB is shown.

Let the projecting plane for the plan ab revolve about this line as an axis, into the horizontal plane.

From points a and b set off the projectors aA and bB , equal to aa' and bb' respectively, and at right angles to ab , on the same side of it.

Join A and B ; AB is the required distance or length of segment.

Observe. The length AB (being the hypotenuse of the right-angled triangle of which the sides are ab and the difference of the projectors Bb and Aa) may be found by

setting this difference from b along bB to B' and joining B' to a .

2. The angle a straight line makes with a given plane is the angle between the line and its orthographic projection on that plane, or the angle which AB makes with ab , and is therefore the angle θ .

For the same reason ϕ is the angle which the given line makes with the vertical plane of projection.

3. The 'traces' of the line being points in the planes of projection have each of them a projection in xy , viz. h' and v (Theorem III.); but the elevation $a'b'$ produced contains h' , therefore the intersection of these lines, namely, xy and the elevation, gives point h' ; and Theorem I. and the plan of the line give point h . The corresponding loci of v and v' determine those points.

The distance between the traces H and V of the given line is obviously the length hV , as shown in figure.

Corollaries given below.

As an exercise on this problem determine the inclinations of the given lines, AB , CD , EF , &c., and their traces, and the lengths of the segments indicated. See Fig. 4.

Converse of Problem I.:

- (1) Given the traces of a straight line; or
- (2) Its inclination and plan; or
- (3) Its length and inclination; or
- (4) Length and position of extremities of finite portion; to draw or determine the projections of the line.

(1) If the traces hh' and vv' be given,—
Join h and v and h' and v' .
Then hv , $h'v'$ are the projections required.

(2) If the plan $h\nu$ be given making a given angle hva with xy , and the line inclined at a angle of θ degrees.

At any point h in the given plan draw hV , making the angle θ° with $h\nu$.

Assume points a and b in the plan, draw the perpendiculars aA and bB , and find points a' and b' by Theorems I. and II. and join a', b' .

Then $ab, a'b'$ are the projections.

(3) If the length AB be given, its inclination θ , and the position aa' of one extremity A .

Through a draw the indefinite plan $h\nu$.

Set aA at right angles to ab and equal to aa' .

Through A draw hV , making the angle θ with $h\nu$.

From A set off AB along this line and determine b and b' . $ab, a'b'$ are the required projections.

(4) If the length AB be given and the positions of its extremities A and B with regard to the two planes.

Draw aa' the projections of one extremity A , from the conditions Theorems I. and II.

Also any point pp' from the conditions for the other extremity B and through p draw a parallel to xy : this is a locus of b the other extremity of the plan ab required. Determine the length ab^* of the plan; measure this length from a to a point b in the parallel, and the position of ab is fixed; b' may be found of the height of p' , and $ab, a'b'$ are determined.

* The base of a right-angled triangle, hypotenuse AB and altitude equal the difference of the heights of A and B .

The student will observe that the problems admit of one, two, or more, or an infinite number of solutions. These cases should be studied. See Problem 2.

I (3), COR. 1. *To determine the 'traces' of a given plane when the data are:*

- (1) *Two straight lines (parallel or meeting).*
- (2) *A straight line and a point.*
- (3) *Three points.*

(1) The horizontal and vertical traces of the given lines must be found by Problem 1, and the corresponding traces of the plane drawn through the points thus found. Theorem VI.

(2) Find the traces of the given line.

Draw the projections of a line parallel to it, through those of the given point (Theorem IV.), and find its traces.

Or, by Theorem I., determine the projections of any point qq' in the given straight line; join the projections of this point with pp' those of the given one; then the line PQ lies in the given plane.

The traces of the two lines give those of the plane.

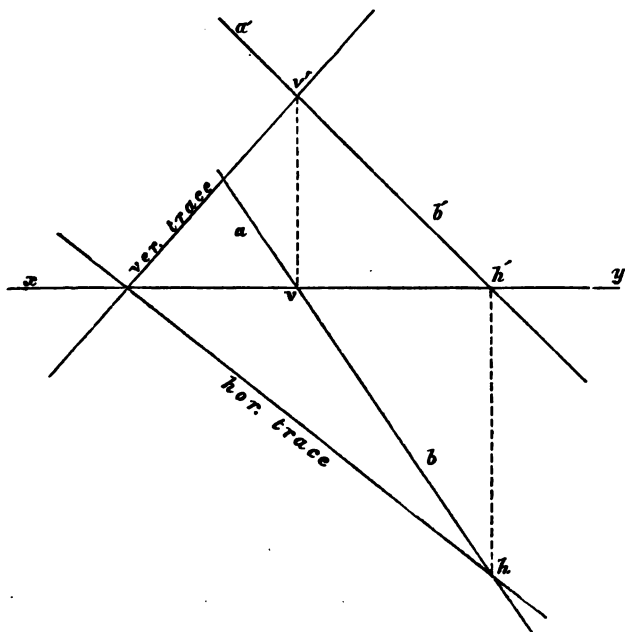
(3) Join the corresponding projections of the given points; and determine the traces of the three lines of which those joining lines are the projections.

The traces of the three lines give those of the plane.

Obs. If the given plane meets xy only three traces of the lines lying in it need be found; e.g. two in the horizontal trace of the plane and one in its vertical trace; the latter joined to the point of intersection of the horizontal trace of the plane with xy gives the vertical trace of the plane.

I. COR. 2 (see figure). *Given the traces of a plane and one projection ab of a straight line which lies in that plane; to determine the second projection $a'b'$ of the line.*

Fig. 7.



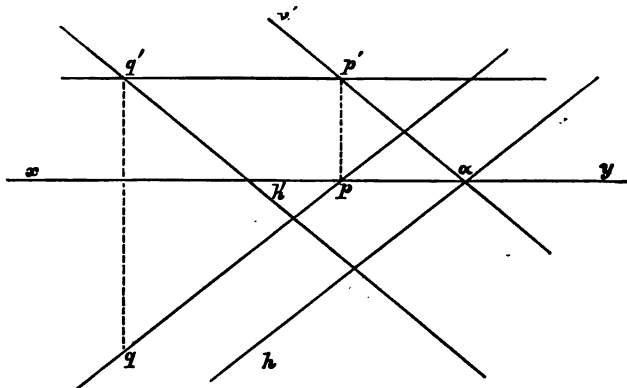
Since the traces hh' and vv' of the line must be *in* those of the given plane (Theorem VI.), determine these points, then the line drawn through points h' , v' is $a'b'$ the required elevation.

Observe. Point H lies in the given straight line AB , hence the projections hh' of the point are in those of the line. Thus ab is a locus of h , and of v ; the horizontal trace

of the plane is a second locus of h , and xy of v , which points are thus determined: perpendiculars to xy through them are loci of h' and v' , Theorem I, which points are thus also determined.

I. COR. 3. *To determine a point pp' of given altitude a inches (or a'') in a given plane hav' .*

Fig. 8.



In the given vertical trace av' , take a point p' , a'' high, Theorem II; find p in xy , Theorem I; then pp' is the required point.

I. COR. 4. *To determine a horizontal line a'' high, in a given plane hav' .* Fig. 8.

Determine pp' as before.

Through p draw pq parallel to ha , Theorem IV.

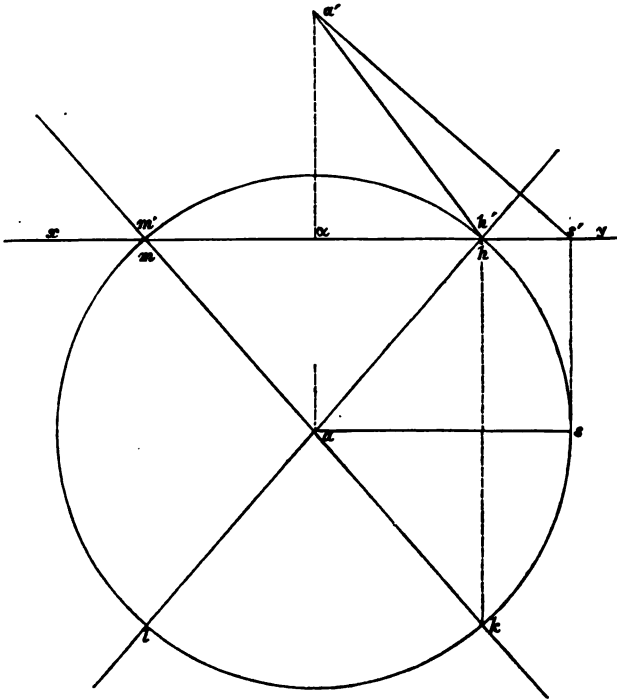
Through p' draw $p'q'$ parallel to xy .

Then $pq, p'q'$ is the required line.

PROBLEM II.

Through a given point aa' to draw a straight line inclined 6° to the horizontal plane.

Fig. 9.



If a right cone be determined having its vertex in the given point A , axis equal to $a'a$ and generatrix inclined θ , every generatrix of the cone fulfils the given

conditions. The conic surface is therefore a locus of the required line.

To work the problem:

Through point a' draw $a's'$ making the angle $a's'a$ equal to θ .

Through a draw as parallel to xy , then $as, a's'$ is parallel to the vertical plane and inclined θ° .

With centre a and radius equal to as' describe the circular base klm of the cone which gives the complete solution.

Observe. The four lines AH, AK, AL, AM are inclined θ° , and make equal angles with the vertical plane.

PROBLEM III.

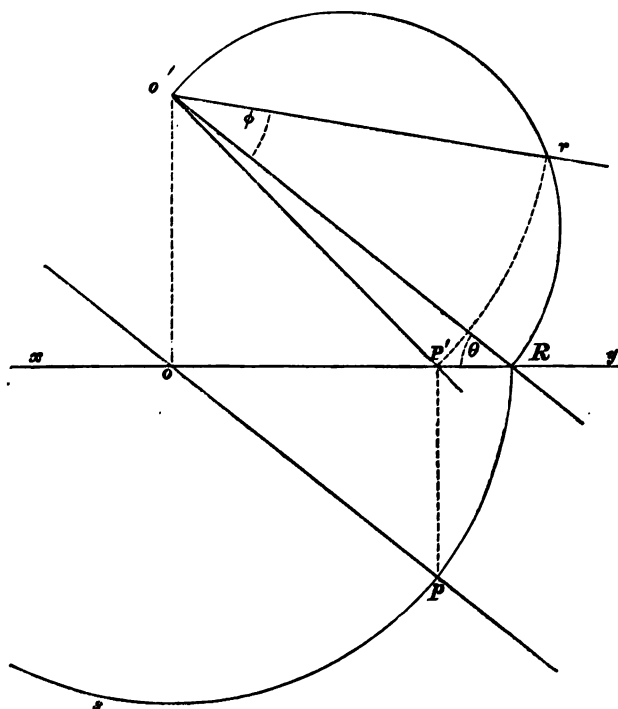
To determine a straight line which shall make a given angle θ with the horizontal plane and an angle ϕ with the vertical plane of projection.

A line inclined θ° will lie on the surface of a right cone with axis vertical and generatrix inclined θ° to the base.

Determine such a cone having its vertex in point oo' in the plane of elevation: Rps will be its circular base. That generatrix of the cone which makes an angle ϕ with the vertical plane will be the line required. To find this. Knowing the length $o'R$ of this generatrix, determine the length $o'r$ of its elevation as in the figure; and since the position of o' one extremity of it is fixed, that of the other, viz. p' , in xy , may be found as indicated. The perpendicular to xy from p' is a locus of p , and intersects the cir-

cular arc Rps , which is another locus of p , which point is thus determined. Join op , $o'p'$; it is the required line.

Fig. 10.



It is obvious that there are four such lines on the surface of the cone, and therefore there are four solutions of the problem, when oo' is a given point in the line.

It will also appear from the above, that the second angle ϕ cannot exceed the complement of θ or $(90 - \theta)$ degrees.

PROBLEM. IV.

To draw or determine the traces of a plane inclined θ degrees and perpendicular to the vertical plane of projection. See Fig. 5, 'Planes'.

Draw the horizontal trace ql of the required plane at right angles to xy , meeting it in point l ; and through l draw lm' making the angle ylm' equal θ : qlm' is the required plane.

It is obvious that lm' and ly are the two perpendiculars to ql , the horizontal trace, or common section of the planes, at the point l in it, which form the measuring-angle of the inclination of these planes to each other.

The plane of lm' and ly having been turned into the horizontal plane, lm' in Fig. is no longer perpendicular to ql .

Similarly (see Fig. 'planes') a plane $n\beta o'$ may be drawn at right angles to the horizontal and making an angle ϕ with the vertical plane of projection.

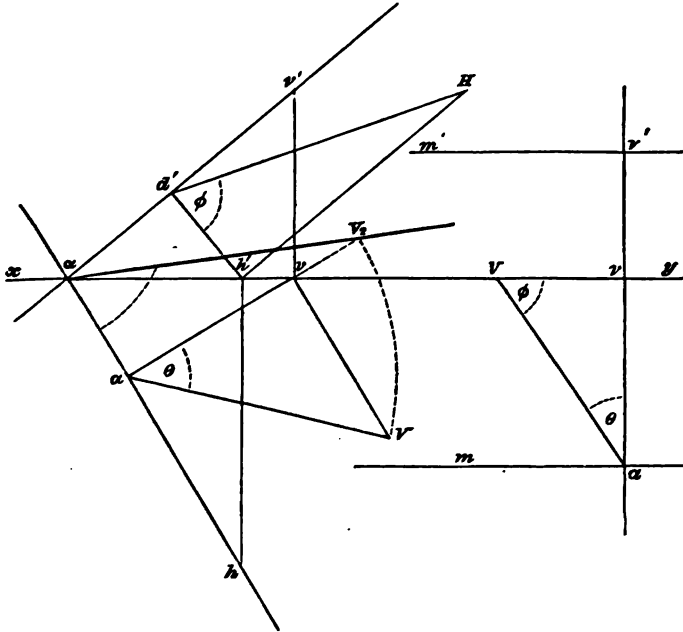
PROBLEM V.

To determine the angles θ and ϕ which a given plane has or makes with the planes of projection; and to determine the angle between its traces.

Since a dihedral angle is measured by the plane angle formed by two perpendiculars drawn one in each plane to the same point in their common section, the plane of this rectilinear angle is perpendicular to the 'trace' of the given plane. 'Construct' the plane of this angle, revolving it about that line of it which is in the plane of projection in question, and the required angle of inclination is obtained.

Thus at point a in the horizontal trace ha (Fig. 11) two such perpendiculars av and aV were found, and at

Fig. 11.



point d' in the vertical trace perpendiculars $d'h'$ and $d'H$, and the required angles shown.

To determine the magnitude of the angle hav' between the traces of the plane.

Let the plane be 'constructed' or folded about its horizontal trace ha until it coincides with the horizontal plane.

Then the length aV measured from a along av produced to V_s gives a second point in the second line of the required angle. Join aV_s , and $\angle aV_s$ is the angle between the traces of the plane.

PROBLEM VI.

Given the horizontal trace ah of a plane inclined θ to determine its vertical trace av' .

At any point a in the horizontal trace draw av at right angles to it and meeting xy in v . Fig. 11. Draw aV making the angle vaV equal to the given angle θ . From v draw vV at right angles to av , and from v set off the length vv' at right angles to xy and equal to vV . Then the straight line through a and v' is the vertical trace required.

See Problem III. Chap. IX.

PROBLEM VII.

To determine the traces of a plane making an angle of θ degrees with the horizontal and an angle of ϕ degrees with the vertical plane.

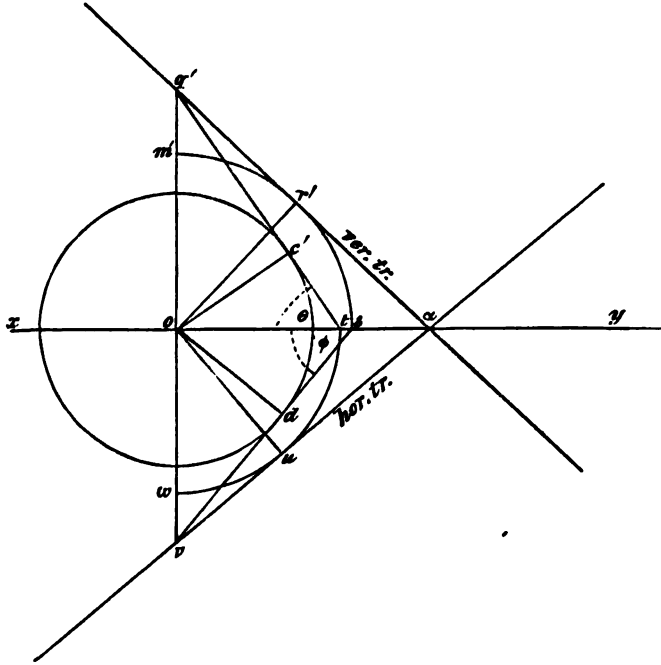
When a plane touches a right cone it is inclined at the angle which the generatrix of the cone makes with its base.

Determine two right cones (Fig. 12) with vertices q' and v and bases tuv and $s'm'$ in the horizontal and vertical planes respectively; the generatrix $q't$ making an angle θ with its base tuv , and generatrix vs an angle ϕ with its base $s'm'$. The common tangent plane to these cones is the plane required.

The surfaces of the cones have a common point pp' which is also on the surface of a common sphere enveloped

by them and having its centre o in xy . See Figure 13. Hence the plane of points P , Q and V is that required.

Fig. 12.

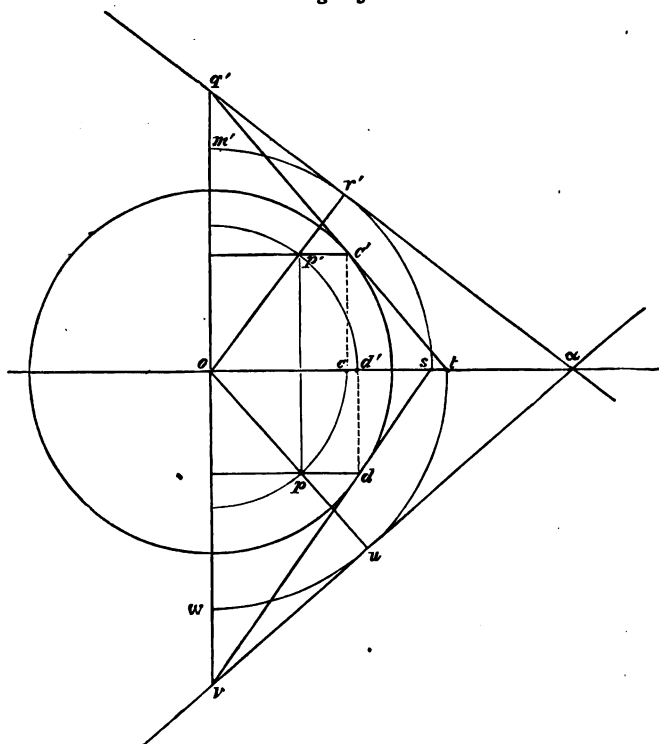


To work the problem.

Commence with one of the vertices, assuming point Q or q' in the vertical plane. Fig. 12. Draw $q'o$ indefinitely at right angles to xy , meeting it in o ; and draw $q't$ as above described, making angle θ with xy , on which let fall the perpendicular oc' from o ; the circle described with centre o and radius oc' gives the common inscribed sphere of the

two cones. vs drawn as above mentioned, making the angle ϕ with xy , and touching this circle in d , meets $q'o$ produced in the point v , which is the second vertex. Tangents to the bases of these cones through v and q' meet xy in point a and give the traces of the required plane $q'av$.

Fig. 13.

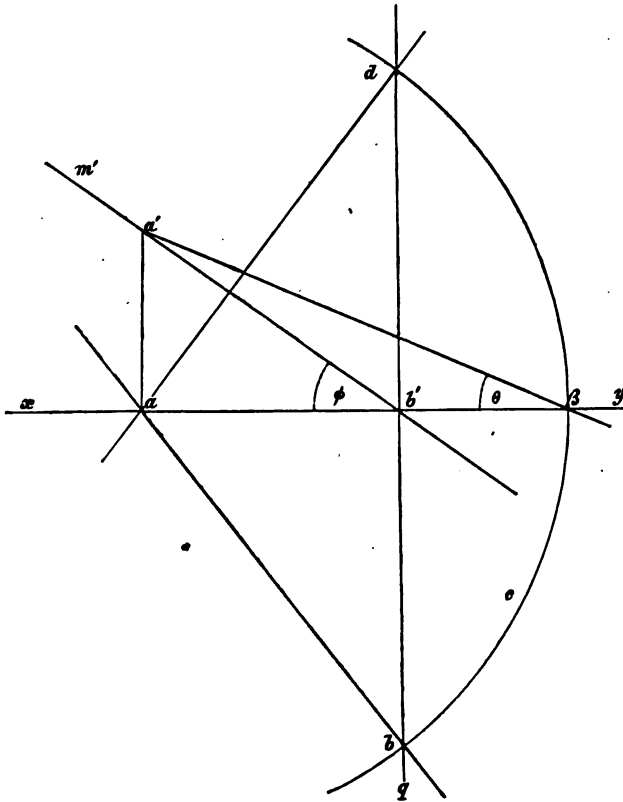


Limits. The sum of the given inclinations or $\theta + \phi$ lies between 90° and 180° .

PROBLEM VIII.

To determine a straight line inclined 6° to lie in a given plane qb'm'.

Fig. 14.



Limits. The angle ϕ is the angle of inclination of the given plane. It is therefore obvious that the angle of inclination θ of the given line must be *not greater than* ϕ . For no straight line can lie in a plane and have a greater inclination than that of the plane.

Construction. Assume point aa' in the given plane.

Make the angle $a'\beta a$ equal θ .

With centre a and radius $a\beta$ describe the arc βcb meeting the trace qb' in b or d ; then ab , $a'b'$ or ad , $a'd'$ is the required line.

Observe, that each of the lines AB and AD satisfies the conditions of the problem.

The cone, vertex A and base dcb , is the locus of all lines through A inclined θ .

The given plane is a second locus of the required line. And these loci intersect in the lines AB and AD only.

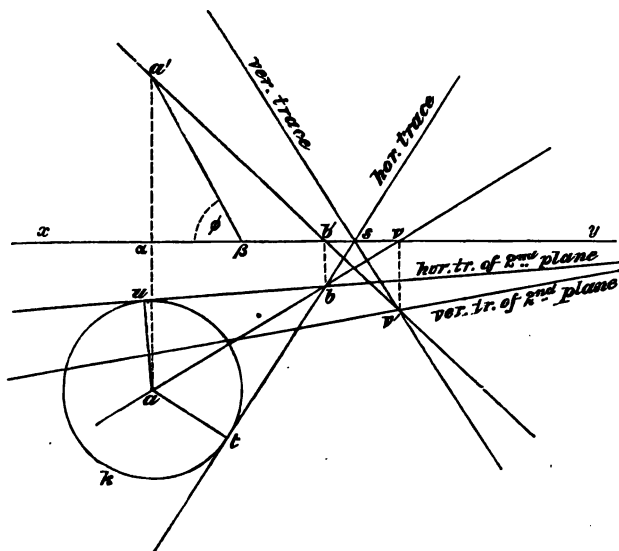
PROBLEM IX (*converse of Problem VIII*).

Through the given straight line ab , $a'b'$ (which is inclined θ) to draw a plane inclined ϕ .

Since the given line is to lie in the required plane, its traces must lie in those of the plane, Theorem VI. Determine these traces bb' , vv' . Problem I.

Determine a right cone, with vertex in point aa' of the given line, and base tuk in the horizontal plane, its generatrix making the given angle ϕ with the base. Every tangent plane to this cone is inclined ϕ , Theorem VII.; and the straight line through b touching the base of the

Fig. 15.



cone in t is the horizontal trace of such plane, Theorem VIII. By producing this line to meet xy in s and joining sv' , we obtain the vertical trace of the required plane.

For this plane tsv' , passing through points A and B in the given line, contains that line; and touching the cone has the required inclination.

Note: That the radius at of the base of the cone is taken equal to $a\beta$ in figure.

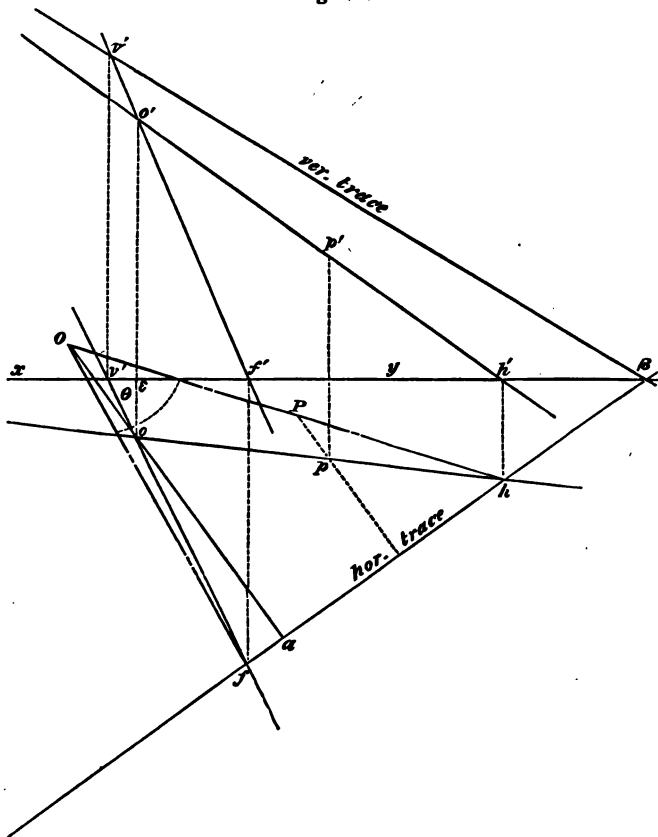
Also, that a second plane bwr' , having ub for its horizontal trace, likewise satisfies the conditions of the problem.

N.B. Point w lies out of the figure.

PROBLEM X.

To determine the magnitude of the angle which the two given straight lines OH, OF make with each other.

Fig. 16.



Determine the horizontal traces ff' and hh' of the given lines, then (Theorem VI.) the horizontal trace $f\beta$ of the

plane of the lines passes through points f and h , and its vertical trace $\beta v'$ through v' , vv' being the vertical trace of OF .

'Construct' the plane $f\beta v'$, revolving it about its trace $f\beta$ into the horizontal plane; then fOh the required angle is exhibited.

The indefinite perpendicular on the horizontal trace through the point o is a locus of O (Theorem X.), and the length Oa is made equal to the hypotenuse of the right-angled triangle, of which oa is the base and $o'e$ the altitude.

COR. To bisect the angle formed by two given straight lines, as OF and OH .

'Construct' the plane of the angle as above; and bisect the angle fOh thus found by a straight Os meeting the horizontal trace used in point s ; obtain s' in xy , and join os , $o's'$; these are the projections of the required bisecting line.

PROBLEM XI. (See Fig. 16.)

To draw a straight line oh , $o'h'$ through a given point pp' to make a given angle θ with a given straight line of, $o'f'$.

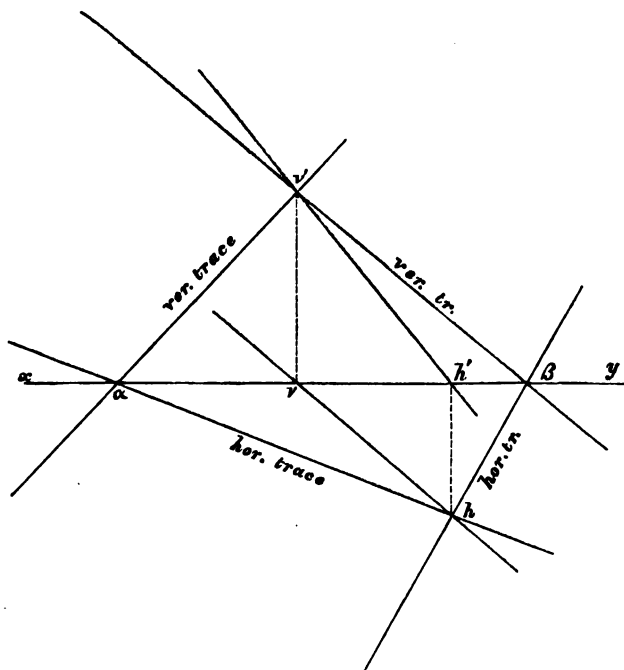
Determine the horizontal trace $f\beta$ of the plane of the given straight line OF and point P (Prob. I, Cor. 1). 'Construct' this plane, and through P so 'constructed,' draw the line OP making the given angle θ with OF in the horizontal plane. Determine the projections oo' of point O , join op , $o'p'$, they are the projections of the required line.

Note: That the construction of this problem is made more simple by using a second plane of elevation perpendicular to that of the angle. Theorem III. (2).

PROBLEM XII.

To determine the projections of the intersection of two given planes, hav' and $h\beta v'$.

Fig. 17.



The planes intersect in a straight line, which lying in both planes has its traces in those of the planes. Hence v' , in which the vertical traces meet, is the vertical trace of the line of intersection, the *plan* v of this point is in xy , and is found by a perpendicular to it (Theorem I.). Simi-

larly, h is the horizontal trace of the line required, and H' , its elevation, is found in xy by a perpendicular to that line.

Join hv , $H'v'$, these are the projections of the required line.

COR. 1. *To determine the intersection of three given planes.*

Determine the intersection HV as in the problem, of planes (1) and (2): then the intersection AB of planes (2) and (3). AB will meet HV in a point P which is the intersection required and is the vertex of the solid angle formed by the three given planes.

COR. 2. *To determine a straight line which shall pass through a given point oo' and meet two given straight lines ab , $a'b'$ and de , $d'e'$.*

Determine the traces of the plane of the point O and the line AB ; likewise those of the plane of point O and the straight line DE . Then the intersection of these two planes is the line required.

PROBLEM XIII.

To determine a straight line at right angles to a given plane qlm' , which shall pass through a given point pp' .
Fig. 18.

Since the projections of a perpendicular to a plane are perpendiculars to the 'traces' of the plane, Theorem V; ps drawn through a point p at right angles to ql , and $p's'$ at right angles to lm' , are the projections of the required line.

Conversely. To determine a plane at right angles to a given straight line ps , $p's'$ which shall pass through a given point ii' in the same.

Through A draw ql at right angles to ps meeting xy in l , and through l draw lm' at right angles to $p's'$: qlm' is the required plane.

COR. 1. *To determine the distance of a given point pp' from a given plane qlm' .*

Before reading this corollary study the following Problem No. XIV., and by it determine point ii' in which the perpendicular PS to the plane intersects it.

Then by Problem I., determine the length IP of the perpendicular of which ip , $i'p'$ are the projections. IP is the required distance of the given point P from the plane.

COR. 2. *To determine two parallel planes at a given distance (n) inches from each other.*

Determine a straight line BC , and in it mark off the length PQ , points P and Q being (n) inches apart; the planes through these points at right angles to the line BC are those required.

PROBLEM XIV.

To determine the point of intersection of the given straight line ps , $p's'$, with the given plane. Fig. 18.

Every plane which contains the given line meets the given plane in a straight line which passes through the required point of intersection of the given straight line and plane. Assume any such plane pmm' , of which pm and mm' are the traces.

Determine the line of intersection hm , $h'm'$ of this assumed plane with the given one.

The projections ii' of the intersection of these two lines HM and PS are those of the required point.

This plane-angle is formed by two straight lines drawn one in each plane from a point in their common section QM at right angles to that line. (Euc. Bk. XI. def. 6). The plane of these perpendiculars is therefore (Euc. Bk. XI. 4) at right angles to the line QM : hence if this plane be determined and 'constructed' about its horizontal trace, the required angle wTr is shown.

To do this, assume the horizontal trace rw of the plane of required angle, at right angles to qm , meeting it in s , and the horizontal traces of the given planes in r and w .

Fold the plane about rw bringing the point T in QM down into the horizontal plane in qm : join Tr , Tw .

The distance Ts is found by making it equal to TS on the vertical plane (as shown), this line being the common section of the 'projecting-plane' of the *plane* qm with the plane of the measuring angle.

Conversely. Given θ the profile or measuring angle of the inclination of two planes to each other, to determine the traces of the planes.

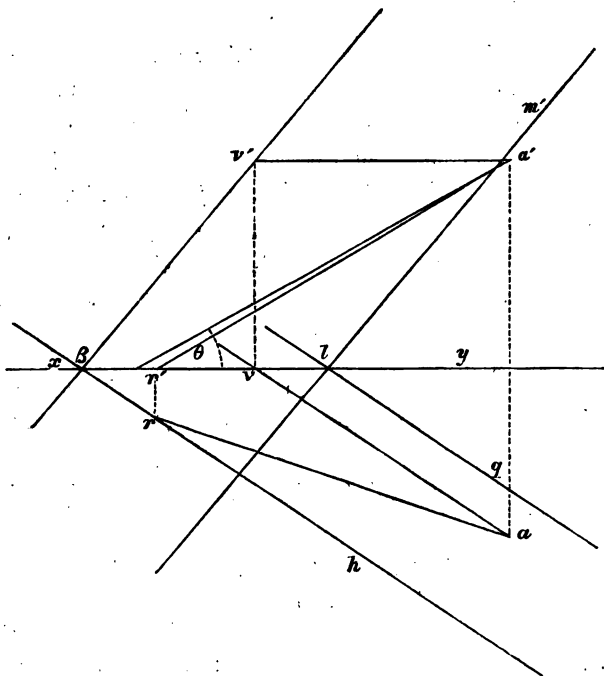
Draw the plane $qq'm'$ and determine the straight line qm , $q'm'$ in it.

Draw rs at right angles to qm and determine sT from ST as before and join rT : then make the angle $rTw = \theta$, and point w is determined, through which draw ql the horizontal trace of the second plane: lm' its vertical trace is drawn through the vertical trace of the line QM (Theorem VI.).

PROBLEM XVI.

Through a given point aa' to draw a plane parallel to a given plane qlm' .

Fig. 20.



Since if two parallel planes be cut by a third plane, their sections with the third are parallel straight lines (Euc. XI. 16), therefore the traces of the given and required planes will be parallel straight lines.

If therefore the given plane be not parallel to xy , one point in either of the required traces being found both those lines can be obtained.

Through the given point A let a straight line be drawn parallel to either trace of the given plane; the trace of this line supplies the required point.

Therefore through A or aa' draw the straight line AV or av , $a'v'$ parallel to ql which is a horizontal line; the trace vv' of this line gives the point v' in the vertical trace $\beta v'$ of the required plane: and βh drawn parallel to ql through β gives the horizontal trace.

PROBLEM XVII.

Through a given point aa' to draw a straight line which shall be parallel to a given plane qlm' and have a given inclination θ . Fig. 20.

Obs. The angle θ cannot exceed the angle of inclination of the given plane.

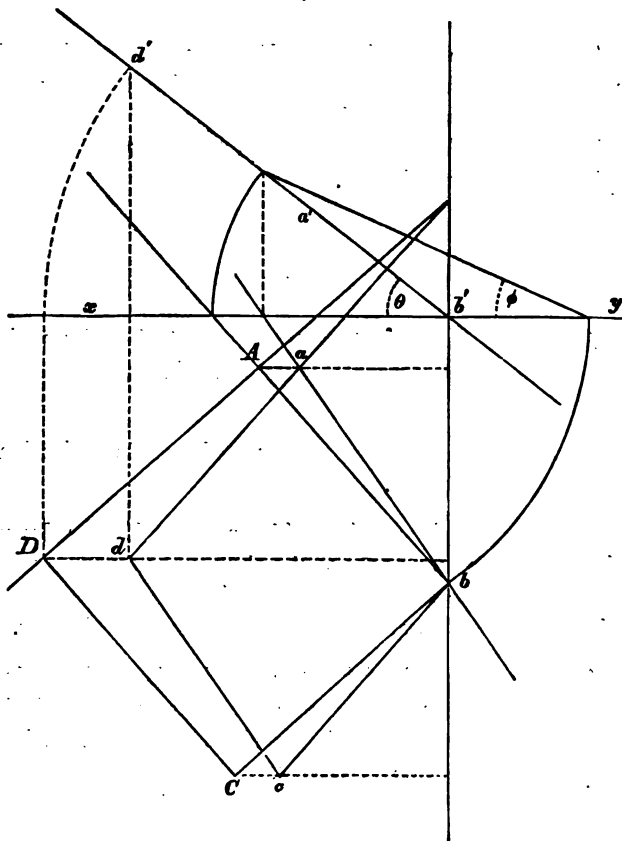
Through the given point aa' let a plane $h\beta v'$ be drawn as in Problem XVI. parallel to the given plane. And through point A draw the straight line ar , $a'r'$ by Problem VIII. to lie in the plane $h\beta v'$, and have an inclination of θ degrees.

Another mode of solving the problem is, to determine in the given plane a straight line having the required inclination, and to draw through the given point A a straight line parallel to the line so formed.

PROBLEM XVIII.

In a given plane $bb'd'$ to determine a straight line which shall be perpendicular to a given straight line $ba, b'a'$ lying in that plane.

Fig. 21.



'Construct' the given plane, revolving it about its horizontal trace bb' into the horizontal plane. Then AB will lie in the horizontal plane in the position Ab . Fig 21.

The point b (or B) does not move during the rotation of the plane, being *in* the axis or trace bb' of the rotating plane. The perpendicular to bb' through point a is a locus of A , and the position of point A *in it* is found by taking the length $b'a$ along the vertical trace $b'a'$ of the plane and setting it off from the line bb' . A and b joined gives the line Ab or AB 'constructed.'

Through point A draw the indefinite line AD perpendicular to Ab , and determine the plan d of any point D in this perpendicular. To do this:—Through D draw a line at right angles to bb' for one locus of d (Theorem X.). Measure the distance of point D from the line bb' and set this length from point b' , along the vertical trace of the plane, to determine d' . A perpendicular from d' to xy is a second locus of d . The intersection of these loci of d determine that point. Join a and d , then ad , $d'd''$ are the projections of the required perpendicular.

Observe. In the same manner a straight line AD may be drawn in a given plane to make any given angle θ with a given straight line AB in that plane.

N.B. Theorems III. and IX. should be studied in connexion with this problem and figure.

PROBLEM XIX.

To determine the projections of a given plane figure, when the inclination of its plane is given and that of a straight line lying in the plane of the figure. Fig. 21.

1st. When the given line forms part of the perimeter of the figure.

Let the given figure be a square, its plane inclined θ degrees and one side AB of the figure inclined ϕ degrees.

Determine the plane $bb'd'$: in it place the line ab , $a'b'$. 'Construct' the plane as in Problem XVIII., and on AB describe the square $ABCD$: revolve the plane back into its former position and obtain $abcd$, the plan of the figure.

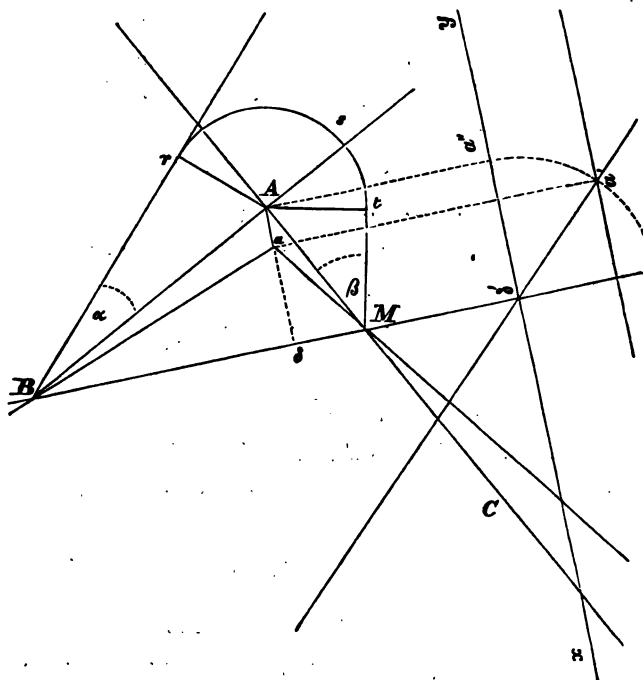
2ndly. When the given straight line is a diagonal of the given figure or its bisecting line: a tangent, a chord, or a diameter of a given ellipse or circle.

Determine the plane $bb'd'$ of the figure and the projection of the given line in it. 'Construct' the plane, and the line as lying in it; draw the given figure in the horizontal plane having the required position with regard to the 'constructed' line: and from thence proceed as before to obtain the projections of the entire figure when turned back into the inclined position.

PROBLEM XX.

Given two straight lines AB, AC making an angle BAC of θ degrees with each other; to determine the projections of the lines when AB is inclined at an angle of α degrees and AC inclined β .

Fig. 22.



Observe the conditions of the problem are impossible if

$\alpha + \beta + \theta$ exceed 180° ;

if $\alpha + \beta + \theta$ equal 180°

the plans of AB and AC will be in one straight line, *i. e.* the plane of the angle BAC will be a perpendicular one.

The case in which $\alpha + \beta + \theta$ is less than 180° will be that here considered.

Draw the angle BAC on the horizontal plane. It will be necessary to revolve the plane of the angle BAC into the inclined position it will occupy when its containing lines have the required inclinations. For this purpose a horizontal line or trace BM of the plane must be discovered, about which the plane is to revolve. Assume a point B in AB and

make the angle $ABr = \alpha^\circ$ and

... .. $ArB = 90^\circ$.

Then Br will be the length of the plan of BA when BA is in the required position.

Now for the same height $Ar (= At)$, by the aid of a tangent tM to the arc rst determine

the angle $AMt = \beta^\circ$ and

... .. $AtM = 90^\circ$: and point M is found.

The straight line BM is the required horizontal trace of the given angle. A plane of elevation having its xy at right angles to this line will be always perpendicular to the plane of that angle; and the elevation of the angle upon it will be in the vertical trace, Theorem III. 2.

The elevation of the path of point A is the circular arc $a''a'$, in which a' is determined by the parallel to xy at the distance Ar .

From a' the plan a is found in $A\delta$ (Theorem I.).

Then BaM is the required plan, and $\delta'a'$ the elevation and vertical trace of the plane of the given angle. $a''\delta'a'$ is the angle of the inclination of the plane of the lines.

COR. To determine three straight lines meeting in a point, each line perpendicular to the other two.

Proceed, as in this problem, to obtain the projections of the two lines AB and AC ; and through point aa' , by Problem XIII., determine the projections ae , ae' of the third line, at right angles to the plane $M\delta a'$. AB , AC , AE are the required lines.

PROBLEM XXI.

To determine the projection of a given plane figure when the heights of three points in the plane of the figure are given.

Limits. When the given heights are equal, the projection of the figure is an equal and similar figure to the given one.

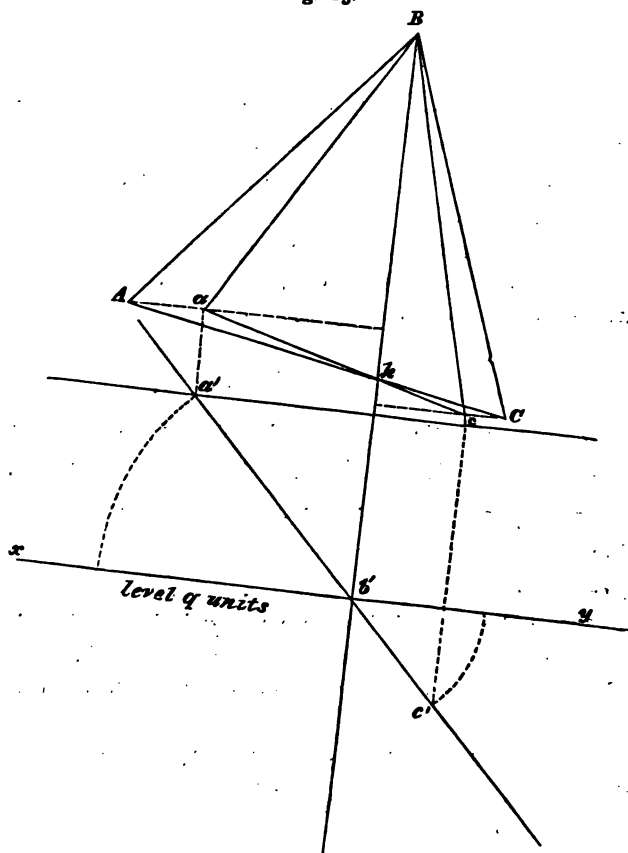
When the given heights are unequal, the problem is a possible one only when the difference between the heights of any two points does not exceed the distance between these points.

If this difference is equal to that distance the plane of the figure is vertical; and the projection required will be a straight line.

Let the three given points A , B and C be at the angles of an equilateral triangle of m inches side, and the heights of A , B and C be p , q and r inches respectively: to determine the plan abc of the triangle. Fig. 23.

(1) Draw ABC in the horizontal plane.

Fig. 23.



(2) Determine point h in AC which will have the same height as B , viz. q inches. Then hB is a horizontal line when the triangle is in its inclined position.

(3) By the aid of an elevation at right angles to hB as shown, the plan abc is determined.

PROBLEM XXII.

To determine two parallel planes which shall contain the given straight lines AB or ab , $a'b'$ and CD or cd , $c'd'$.

1st. If the given lines AB and CD are parallel.

Determine the horizontal traces of the lines; parallel lines through these will be the horizontal traces of the parallel planes required, whence the vertical traces can be obtained.

2ndly. If the given lines AB and CD be neither parallel nor in the same plane.

Through any point A in AB draw a straight line AQ parallel to CD .

Determine the horizontal traces of the lines AB and AQ ; the straight line drawn through these points is the horizontal trace of one of the required planes.

Determine the horizontal trace of CD ; through this point a parallel line to the trace just found is the horizontal trace of the second plane required: and the vertical traces of the two planes can be obtained.

3rdly. If the given lines be not parallel but in the same plane, the problem is impossible.

PROBLEM XXIII.

To determine a straight line at right angles to each of two given straight lines AB and CD .

1st. If the given lines be parallel.

Any straight line drawn in the plane of the parallels perpendicular to one of the lines is also perpendicular to the other.

2ndly. When the given lines are neither parallel nor in the same plane.

From any point P in AB draw a line PQ parallel to CD : then the plane of AB and PQ is parallel to CD .

From any point C in CD draw CS perpendicular to the plane of APQ , meeting it in S . A parallel to PQ or CD through point S meets AB in H . Then HT , parallel to CS from H , meets CD in point T and is the required perpendicular to the given lines.

PROBLEM XXIV.

To determine the angle of inclination θ of a given straight line ab , $a'b'$ to a given plane qlm' . Fig. 24.

In this problem it is convenient to determine the complement of the angle required.

This complementary angle is the angle between the given straight line and a line meeting it in point A and perpendicular to the given plane.

Determine this perpendicular ah , $a'h'$, by Problem XIII., and find its horizontal trace hh' ; likewise bb' that of the given line; the line hb through these points is the horizontal trace of the plane of the lines. 'Construct' their plane about hb bringing A into the horizontal plane. Ah , Ab , when drawn, give bAh the complementary angle or $90 - \theta$, and θ is the angle sought.

Conversely:

To determine a straight line through a given point aa' , to make a given angle θ with a given plane qlm' .

Through aa' draw ah , $a'h'$ at right angles to the given plane, and determine its horizontal trace hh' . Through

$90 - \theta$ with Ah meet the trace hb in point b . Then b' being found in xy and joined to a' , the required line ab , $a'b'$ is determined.

Observe. An infinite number of such lines ab , $a'b'$ can be determined to satisfy the given conditions; and their 'locus' is the right-conical surface having its circular base in the given plane and its generatrix inclined θ degrees to the same.

PROBLEM XXV.

To determine two planes which shall be perpendicular to each other and inclined at angles of θ and ϕ degrees respectively to the horizontal plane.

It will be obvious that the angle of inclination of the second plane cannot be less than the complement of that of the first; or ϕ not less than $90 - \theta$.

Therefore

if ϕ is greater than $90 - \theta$, there are two solutions:

... ϕ is equal to $90 - \theta$, there is one solution;

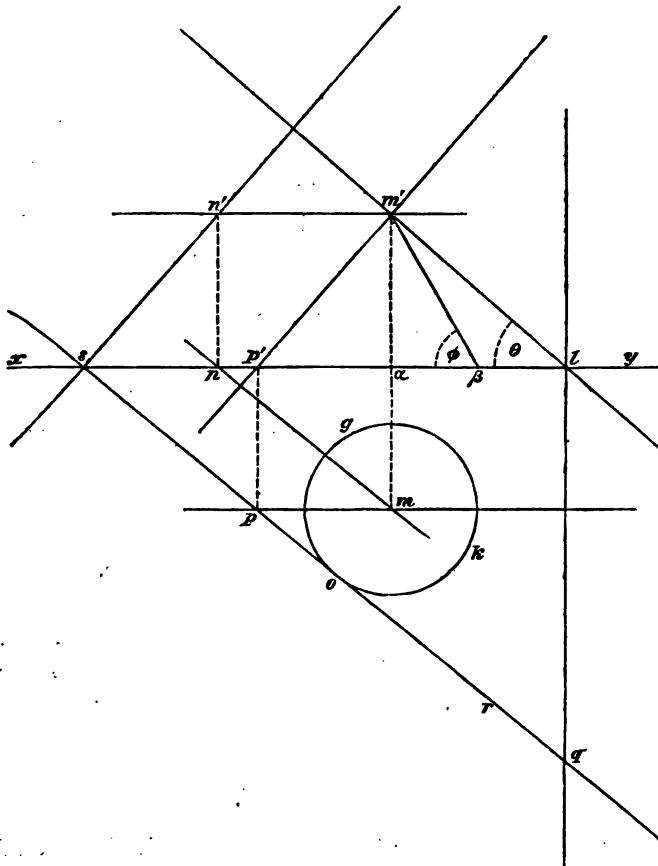
... ϕ is less than $90 - \theta$, there is no solution.

This will appear by what follows.

Draw the plane qlm' inclined θ degrees; also the straight line pm , $p'm'$ at right angles to it (Problem 13), and find the horizontal trace pp' of PM .

Then every plane which contains that line is at right angles to the plane θ (Eucl. XI, 18). Wherefore if such a plane revolve about PM , as an axis, until it makes an angle ϕ with the horizontal plane, it is the plane required,

Fig. 25.



Let it therefore be made to touch a right cone, vertex in any point mm' in PM and generatrix $m'\beta$ making an angle of ϕ degrees with its base gko . The horizontal trace of this plane must touch gko ; (Theorem VII.).

And since the plane is to contain PM , the horizontal trace rp of the plane must pass through p , (Theorem VI.).

Through the point mm' draw the horizontal line mn , $m'n'$ parallel to the trace rp , and determine its vertical trace nn' .

Through n' and s draw the vertical trace of the plane.

Observe that the plane rsn' by containing PM is perpendicular to plane θ ; and, by touching the cone described, has the required inclination ϕ ; it thus satisfies the conditions of the problem.

Note also that the inclination of PM is $90 - \theta$; since if a straight line and an inclined plane be perpendicular to each other their inclinations are complementary.

Cor. To determine three planes each perpendicular to the other two.

For the first and second, determine planes qlm' and rsn' as in the above problem.

Find their intersection mq , $m'q'$ by Problem 12.

The plane which is perpendicular to MQ is perpendicular to the planes which meet in this line (Eucl. XI. 19). Therefore such plane, determined by Problem 13, is the 3rd plane required.

PROBLEM XXVI.

To determine a plane inclined at an angle of θ degrees and making a given angle α with a given inclined plane.

Let qlm' be the given inclined plane.

To fulfil both conditions, the required plane must touch two right cones having a common vertex in any point vv'

* δ' is the point of intersection of the lines $v's'$ and lm' and is not lettered in the fig.: also the plan of the circular base $\delta's'$ is not shown in the fig.

by Problem 12.

The line MQ is perpendicular to the line EF . (See Prob. 13.)

PROBLEM XXVI.

Two planes inclined at an angle of θ degrees and
cut a right circular cone.

From either of the points, as B , draw the straight line BR at right angles to the given plane (Problem 13), meeting it in S , determined by Problem 14.

Make $SR = SB$, point R being on the opposite side of the plane to B ; join R, C , cutting the given plane in point Q : QB, QC are the required lines.

EXERCISES ON CHAPTER I.

1. Determine the projections of four points, each 2 inches from the planes of projection, viz. one in each dihedral angle.

2. Determine four points each in a plane of projection and 3 inches from xy ; no two of them to be on the same side of that line.

Supposing the loci of these points (Theorem I.) to be 2 inches apart, determine the distance between any two points.

3. A point is 2.5 inches from the plans of projection, draw two lines through it inclined 50° ; their plans making angles of 35° with xy . Problem 1, converse (2).

Determine the traces of the lines: also the angle which the lines make with each other. Problem 10.

4. Through a point 2 inches from each plane of projection draw lines inclined 30° , making angles of 40° with the vertical plane. Problem 3 and Theorem IV.

5. A straight line in the horizontal plane makes an angle of 45° with xy and is a trace of planes which are inclined 50° ; find their vertical traces. Problem 3, "tangent planes to cones," Chap. ix.

6. A regular hexagon, side 2 inches, has its diameter and adjacent side inclined 30° and 45° respectively: draw its plan and determine the inclination of its plane. Problem 20.

7. Draw two parallel planes 2 inches apart and inclined 60° . Problems 1 and 13, and Problem 25, *note*.

8. A plane is inclined 40° , and is at right angles to another which is inclined 70° : draw the traces of the planes. Problem 25.

9. The horizontal traces of two planes, inclined 45° and 30° respectively, make an angle of 60° : determine a third plane at right angles to the given ones. Problems 12 and 13.

10. Two planes are inclined 60° and 70° respectively and make an angle of 50° with each other: determine their vertical traces on a plane not perpendicular to them.

11. A regular pentagon, side 2.5 inches, has three of its angular points respectively 1, 2, and 3 inches high. Draw its plan. Problem 21.

12. A square, side 3 inches, lies in a plane inclined 45° , one side of the figure makes an angle of 40° with the horizontal trace of its plane. Draw its plan. Problem 18.

13. A plane is inclined 80° and makes an angle of 60° with a plane which is inclined 50° . Draw the traces of the planes. Problem 26.

14. The two traces of a plane make an angle of 50° with each other and make equal angles with xy : determine these latter angles and the inclination of the plane.

15. Three straight lines are perpendicular to one another and two of them are inclined 25° and 40° : determine the inclination of the third line. Problems 20 and 13.

16. The horizontal and vertical traces of a plane make angles of 30° and 55° respectively with xy ; a straight line parallel to xy is 2 inches from the vertical and 3 inches from the horizontal plane: find the intersection of the given line and plane. Problem 14.

17. Two planes contain a right angle; one of them is inclined 60° and their intersection is inclined 50° . Draw the traces of the planes and find the inclination of the second. Problems 8 and 13.

18. The hypotenuse of a right-angled triangle is 4 inches long, and horizontal; the plans of the sides make an angle of 115° with each other: determine the lengths and inclinations of the sides. Can more than one set of answers be given? Eucl. III. 33, and Problem 20.

19. A straight line inclined 40° lies in a plane inclined 60° ; determine a plane containing the given line and perpendicular to the given plane. Problems 8 and 13.

20. Through a point 3 inches high draw three planes, inclinations 35° , 45° , and 60° respectively; forming a trihedral angle at the given point. Show the real magnitude of the angle of each face of the solid angle. Problems 12 and 10.

MEMORANDA

FOR WORKING THE PROBLEMS IN ENSUING CHAPTERS.

All the solids given are assumed to be '*right*,' unless otherwise expressed.

All dimensions and arrangements, not expressly mentioned and limited, may be assumed at pleasure.

The inclinations of all lines and planes must be understood, unless otherwise mentioned, to be *to the horizontal plane*. So likewise with all '*constructions*;' see page 7.

Invisible edges. In *plan* the invisible edges are those which are hidden by the rest of the solid to the eye looking through the solid at right angles to *the horizontal plane*. In *elevation* they are those which are similarly concealed from the eye looking through the solid at right angles to *the vertical plane*. Invisible edges must be drawn in dotted lines.

Sections. The teacher should give directions for sections with each problem, as far as possible.

II.

SOLIDS IN SIMPLE POSITIONS.

1. *Draw plan and elevation of a pyramid, 3·5 inches high, with square base of 2·5 inches side, when resting with its base on the horizontal plane, and with one side of the base making an angle of 30° with the vertical plane.*

Commence with the *plan*. This will be the square of the base with the opposite corners joined for the plans of the slant edges of the solid. Take the ground line xy inclined 30° with one side of the square and draw perpendiculars to xy (Theorem I.) from the four corners and the centre of the square. The elevations of the four corners of the square will be in the ground line, because the base of the solid rests on the horizontal plane. The height of the pyramid, 3·5 inches, must be set up from the point where the perpendicular from the centre of the square meets the ground line, and at right angles to the ground line (Theorem II.). The whole elevation will be completed by joining the elevation of the vertex to the four points determined on the ground line for the elevations of the four corners of the base.

Invisible edges. In this *plan* all the edges of the solid are visible. In this *elevation* the slant edge of the pyramid nearest xy is invisible. The two sides of the base nearest xy are also invisible, but as they coincide with the two other sides cannot be shown in dotted lines.

NOTE. It must be understood that from any one *plan* and *elevation* any number of elevations can be drawn by assuming vertical planes, that is, by taking new ground lines, in different positions round the solid, and working from the *plan* to them. For example, if a solid were placed on the floor of a room, four elevations of it might be shown on the four walls of the room which are so many vertical planes of elevation. And so from any one *elevation* and *plan* any number of plans may be determined by assuming horizontal planes in the desired position about the elevation, and working from the *elevation* to them.

In this and nearly all examples new ground lines for new elevations of the object should be given by the teacher.

N.B. *For a new elevation.* The new ground line, xy , being given, draw perpendiculars to it from the *plans* of the various points of the solid, and from this xy mark on these perpendiculars the various heights of the points, to be taken from the elevation already drawn, and complete by joining the points thus found with their proper lines.

For a new plan. Draw perpendiculars to the new xy from the elevations of the various points, and mark from this xy on these perpendiculars the various distances which the points are from the plane of elevation in the *plan* already drawn, and complete by joining the proper points.

2. *Plan and elevation of a pyramid with base a regular hexagon of 1.5 inches side, when resting on its base, and with one side of base inclined 40° to the vertical plane. Height 3.5 inches.*

3. *Plan and elevation of a prism, 3.5 inches long, with square bases of 2.5 inches side, when resting on one base and with one face inclined 30° to the vertical plane.*

From its position a square of 2.5 inches will represent the whole solid in plan. The sides of the square will be the plans of the vertical faces; if therefore xy be taken at 30° with one of these sides, the required position of the vertical plane will be secured.

4. *Plan and elevation of a cylinder; axis 3.5 inches, horizontal, and inclined 40° to the vertical plane. Diameter of base 2.5 inches.*

The plan of the cylinder will be a rectangular parallelogram. In elevation the bases will be two ellipses, which will be best drawn by determining their major and minor axes from the plan. To do this, determine the elevations of the four points which are at the extremities of the horizontal and vertical diameters of each circular base: these points will be the extremities of the major and minor axes of the required ellipses,

5. *Plan and elevation of a cone standing on its base. Height 3.5 inches. Diameter of base 2.5 inches. Determine also a second plan when resting on its conical surface,*

For the second plan, take the plane of elevation determined and draw its new xy parallel to one of the sides of the triangle.

6. *Plan and elevation of a prism, 3.5 inches long, with bases regular hexagons of 1.5 inches side, when its lowest face is inclined 20° , its axis horizontal, and inclined 40° to the vertical plane.*

Commence by drawing an *elevation*, when the solid is so placed that the elevations of both bases coincide, or when the axis is at right angles to the plane of elevation. This elevation will be simply the regular hexagon. A *plan* can readily be deduced from it by taking a ground line making an angle of 20° with one of the sides of the hexagon. A second elevation, the one required in the question, can then be determined by taking another ground line at 40° with the plan of the axis. The heights in this second elevation can be taken from the first auxiliary elevation.

7. *Plan and elevation of the prism, Problem 3, with its axis horizontal, inclined 45° to the vertical plane, and its lowest face inclined 30° .*

The work corresponds to that in the last Problem.

8. *Plan and elevation of a tetrahedron with one face horizontal and 1 inch above the horizontal plane. Edge 3 inches. One horizontal edge to be inclined 15° to the vertical plane.*

The plan will be the same at whatever height the horizontal base is assumed.

The base being horizontal, its elevation will be a straight line and at the required height, 1 inch, above the ground line.

The edge of the tetrahedron being known, the height of the vertex above the horizontal base can be determined by making a vertical section through the vertex and one edge.

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To find this, make a right-angled triangle with the length from a corner of the base to its centre for the base of the triangle, and the true length of the edge of the tetrahedron for the hypotenuse. The perpendicular will be the height required.

III.

COMBINATIONS AND GROUPS OF SOLIDS.

1. *A prism, with bases equilateral triangles, is placed with one face on the horizontal plane. On it rests a brick, 9 by 3 by 4.5 inches, with one of the short edges of a large face touching the ground, with the centre of that face touching the highest edge of the prism and with the face itself inclined 45° . Horizontal edge of prism 9 inches. The side of the equilateral base to be determined from the question. Scale $\frac{1}{4}$. Draw plan and elevation, and a second elevation taking the vertical plane at 30° with any horizontal edge of the prism.*

Commence with an *elevation* so placed that both bases of the prism coincide. To do this, draw first the elevation of the brick, which will be simply a rectangular parallelogram. Through the lowest corner take xy making the required angle with the line that represents the face inclined 45° , and make the centre of this line the vertex of the triangle, recollecting that the elevation of the base of that triangle must according to the conditions be in xy , the ground line. The work for the *plan* is simple. The second elevation will be determined by taking the new ground line 30° with any horizontal edge of the prism, and working to it.

2. *Instead of the prism take a cylinder with brick touching it and otherwise disposed as above. The base of the cylinder to be determined from the question; i.e. so that the bisecting line of the face of the brick may be in contact with the cylinder.*

3. *Plan and elevation of a block and prism under the following conditions:*

Block, 1 inch thick, with faces 3 inches square and vertical.

Prism, base a regular hexagon of 1 inch side, edges of the faces 4 inches extending equally on each side of the block, and at right angles to its square faces, axis of prism passing through the centre of the block, two faces of prism horizontal.

(1) *Plan and elevation when the block stands with one narrow face on the horizontal plane and with the short edges of that face inclined 30° to the vertical plane.*

Commence with an elevation so placed that both bases of the prism coincide. The elevation of the block will then be the square, and that of the prism the regular hexagon in the centre of this square with two sides parallel to two sides of the square. From this auxiliary elevation determine plan, and thence required elevation.

(2) *Plan and elevation when the whole is tilted so as to rest on one edge of the block and on one corner of the base of the prism.*

If a ground line is taken touching one corner of the block and one adjacent corner of the prism in the *plan* determined above, this plan may be considered as an *elevation* of the solids in the required position and the required plan be at once deduced therefrom by working to the ground line so taken.

4. *Plan and elevation of a cylinder, 4 inches long, diameter of base 2 inches, passing through the centre of a circular slab, 1 inch thick and 3 inches in diameter; the axis of the cylinder at right angles to and extending equally from both faces of the slab. The whole to be placed resting on the rim of the slab and the rim of the cylinder.*

5. *Describe three circles with radii 1.5 inches touching one another and join their centres. Consider the circles as the plans of three spheres resting on the horizontal plane, and the triangle as the plan of the base of a tetrahedron resting upon them. Complete plan and elevation of the whole, and draw a second plan when one of the slant edges of the tetrahedron is horizontal.*

The second plan required should be determined from an elevation, in which one of the slant edges of the tetrahedron is shown in true length. The ground line for this plan should be taken parallel to this edge in elevation.

6. *Draw an equilateral triangle of 1.5 inches side, and on its sides describe squares externally. Consider these squares as the plans of three cubes, and place on the top of them a pyramid 3 inches high, axis vertical and over the centre of the triangle, base an equilateral triangle with sides parallel to those of the former triangle and covering exactly half of the cubes.*

(1) *Draw plan and elevation with the cubes assumed as resting on the horizontal plane, and with no horizontal edges parallel to the vertical plane.*

(2) *Draw plan of the whole when one slant edge of the pyramid is horizontal.*

For arrangement compare directions for the second plan in Problem 5.

7. *A circular slab, 1 inch thick, diameter 3 inches, rests on one face. On it is placed a tetrahedron of 2 inches edge, axis vertical and over the centre of the slab. Draw plan and elevation, and a second plan where the rim of the slab and the vertex of the tetrahedron touch the horizontal plane.*

For the second plan take xy touching the elevations of the vertex and the rim of the slab.

8. *A rectangular prism, 4 inches long, with square bases of 2.5 inches side, is to be drawn when one diagonal of the solid is either vertical or horizontal.*

Draw a plan and elevation of the solid resting on one of the square bases, with one diagonal of the square at right angles to the ground line.

If two opposite corners of the rectangle which is the elevation of the solid be joined, this line will be the elevation of the *diagonal of the solid* shown from its position in its true length. A plan may then be determined with this diagonal vertical *or* horizontal by taking xy at right angles to *or* parallel with the elevation of this diagonal.

9. *Plan and elevation of a pyramid with hexagonal base.*

- a. *When resting on one face ABV.*
- b. *When one face is vertical.*
- c. *When one edge is horizontal.*
- d. *When one edge is vertical.*

Determine plan of the pyramid when resting on its base.

For a. Determine elevation on a vertical plane taken at right angles with the plane of face *ABV*. Then the required plan may be deduced by taking xy parallel to the line which represents the elevation of face *ABV*, and working to it.

For *b*. Take *xy* perpendicular to the line representing the elevation of face *ABV*.

For *c* and *d*. Determine an elevation where the edge is shown in its true length. Then *xy* taken parallel for *c*, perpendicular for *d*, to the elevation of this edge will place the pyramid in proper position for the plans required.

10. *Plan and elevation of an octahedron, edge 3 inches.*

(a) *When an axis of the solid is vertical and rests on the horizontal plane.*

(b) *When resting with one face on the horizontal plane.*

(a) Describe a square of 3 inches side and join the diagonals. This will be a *plan* of the solid in required position. For the *elevation* it is only necessary to know that the length of the axis is equal to a diagonal of the square, and that the heights of the horizontal edges are half that diagonal.

(b) Describe an equilateral triangle of 3 inches side, and from its centre with radius to one of the corners describe a circle. In this circle complete the hexagon, of which the three corners of the triangle are alternate points. This hexagon with the alternate points duly joined is the required *plan*.

For the *elevation* it is necessary to find the distance between the parallel faces. This is done by a vertical section through one axis of the solid. This section is a rhombus, the shorter diagonal of which is an edge of the solid, and the side of which is the altitude of the equilateral triangle forming the horizontal or any other face of the solid. The distance between the opposite sides of the rhombus gives the distance between the parallel faces of the solid.

IV.

SOLIDS WITH THE INCLINATIONS OF THE PLANE OF ONE FACE AND OF ONE EDGE IN THAT FACE GIVEN.

BEFORE commencing this chapter the student should refer to Chapter I., Problem 19.

1. *Plan and elevation of a pyramid, 4 inches high, with square base of 2.5 inches side, when its base is inclined 50° , and no side horizontal.*

(1) Determine by its traces a plane 50° at right angles to the vertical plane. Its horizontal trace will therefore be at right angles to, and its vertical trace make 50° with, the ground line.

(2) "Construct," that is, fold this plane on its horizontal trace into the horizontal plane, and draw the square as then in the horizontal plane.

(3) Lift the plane back to the required position and determine required plan and elevation of the square base. Remember that this plane being at right angles to the vertical plane will have all points in it shewn in elevation in the vertical trace (Theorem III. 2). The square base will therefore have its *elevation* wholly in the vertical trace. To

find it, take the distances from the corners of the square to the horizontal trace when the plane is folded down, and set them along the vertical trace from its junction with xy . This will give these points in *elevation*. Their *plans* will lie in the perpendiculars to xy from their *elevations* (Theorem I.) and also in the perpendiculars to the horizontal trace from the points as "constructed" (Theorem X.). Thus let the point of which a' is the elevation when folded down or 'constructed' be marked A . A perpendicular from a' to xy contains the plan a , as also does the perpendicular from A to the horizontal trace. The plan a therefore lies in the intersection of these perpendiculars; in other words, these perpendiculars are loci of the point a , and their intersection gives the point itself.

(4) The axis will be drawn from the centre of the base in plan and elevation at right angles to the traces, and being parallel to the vertical plane will be set in true length *in elevation*. It only remains to join the proper points and the projections of the solid are completed.

2. *Plan and elevation of a cube of 2.5 inches edge, when one face is inclined 60° , and no side horizontal.*

3. *Plan and elevation of a pyramid, 4 inches high, with regular hexagonal base of 2.5 inches side inclined 70° , no side horizontal, and its lowest corner 1 inch above the ground.*

4. *Plan and elevation of the pyramid in the last problem when the base is inclined 70° and one side of the base 30° .*

In this problem there is given not only the inclination of the plane of the base, but also of one edge in that base.

LIMITS. The problem is therefore to draw a line of given inclination in a plane of given inclination, and this is always possible if *the inclination of the line is not greater than that of the plane.*

(1) Determine by its traces the required plane inclined 70° and at right angles to the vertical plane. Assume any point A in the plane; its elevation a' is by the position of the plane in the vertical trace; its plan a on a perpendicular to xy from a' , and for convenience it may be assumed at the point where this perpendicular meets xy ; and from a' draw $a'm$ to meet the ground line at the given angle 30° . Rotate the triangle thus formed on the vertical line $a'a$, the projector of the point A , until the point m meets the horizontal trace. As A and m are now both in the given plane, the line joining a and m will be the plan of Am and will fulfil the conditions by being the plan of a line wholly in plane 70° and itself inclined 30° .

(2) 'Construct' plane 70° on its horizontal trace into the horizontal plane. On the line Am , thus brought into the horizontal plane, describe the required hexagon, and complete plan and elevation of the whole as in former problems.

5. *Plan and elevation of a cube of 2.5 inches edge when the plane of one face is inclined 65° and one diagonal of that face 25° .*

6. *Plan and elevation of a tetrahedron of 3 inches edge with one face 70° and one edge in that face 40° . Determine also the inclinations of the other edges of the solid.*

7. *Plan and elevation of hexagonal right prism when the plane of one of its faces $ABCD$ is inclined 50° and the edge AB inclined 35° .*

(1) Determine the plan of rectangular face $ABCD$ as in preceding problems.

(2) It will next be necessary to determine the plane of the base, which, by the definition of the solid, is perpendicular to the plane of the face. The line AB already determined is in the base, being common to both planes, and its horizontal trace will therefore be a point in the horizontal trace of the required plane of base. And as the projections of a perpendicular to a plane are perpendicular to its traces, the horizontal trace of the plane of the base may be at once drawn through this point at right angles to the long edges of the prism, that is, in this case to bc or ad .

(3) Take a ground line at right angles to the horizontal trace of plane of base, and set up the height of A or B (both known points) in the new elevation. By the position of the new plane of elevation, the vertical trace will contain the elevations of both A and B , and therefore can at once be drawn.

Fold the plane of the base about its horizontal trace to bring AB down into the horizontal plane. On the AB thus constructed describe hexagon and complete as in former problems.

V.

SOLIDS WITH THE INCLINATIONS OF TWO ADJACENT EDGES GIVEN.

LIMITS. It must be remembered that *the sum of these inclinations with the angle contained by the two edges must not exceed 180° .*

For these problems refer to Chapter I., Problem 20.

1. *Plan and elevation of a cube when the inclinations of two adjacent edges as BA , BC , 20° and 48° are given.*

(1) Draw the square $ABCD$ to represent the face 'constructed' into the horizontal plane. Then to find a horizontal in the plane of this face. At any point E in BA , or BA produced, make the angle BEF 20° . Then if a perpendicular be drawn from B to EF , it will be the length of the line expressing the height to which B will be raised above horizontal plane containing point E when in required position. From B as a centre with this perpendicular for radius, describe a circle. The tangent to this circle to make an angle of 48° (i.e. the inclination of the other edge) with BC will give a point H where it meets BC in the horizontal required. E being another point in it, the straight line through H and E will be the horizontal sought in plane of face $ABCD$.

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(2) Assume a plane of elevation at right angles to this inclined plane. The height to which point B is to be raised being known, a line of level that distance above xy will contain the elevation of B . The rest of the work is similar to that in preceding problems.

NOTE. To explain the principles of this construction, the teacher had better cut a card-board model, by which to illustrate the folding up and down of the plane of the angles and of the lines.

2. *Plan and elevation of a tetrahedron when two of the adjacent edges in one face are inclined 28° and 62° .*

3. *Plan and elevation of hexagonal right prism when the two edges AB , BC in one face are inclined respectively 25° and 40° .*

Determine the plan of the rectangular face $ABCD$ as in Problem 1, and refer to explanations of Problem 8, Chapter IV., to find the plane of base and complete the projections of the solid.

VI.

SOLIDS WITH THE INCLINATIONS OF TWO ADJACENT FACES GIVEN.

FOR these problems refer to Chapter I., Problem 25.

LIMITS. Case 1. If the given planes are at right angles to each other, their inclinations being θ° and ϕ° , ϕ° must be not less than $90^\circ - \theta^\circ$.

Case 2. If the given planes are not at right angles to each other, refer to Chapter I., Problem 26.

1. *Plan and elevation of a cube of 3 inches edge, when the inclinations, 45° and 60° , of the planes of two adjacent faces are given.*

(1) Determine the traces of plane 45° assumed at right angles to the vertical plane.

(2) Determine the projections of a perpendicular to plane 45° , in order to determine plane 60° , since every plane containing this perpendicular is at right angles to the former plane.

(3) Determine *horizontal trace* of plane 60° . To do this, make the point at which the perpendicular pierces

plane 45° the vertex of a right cone, generatrix 60° , base in horizontal plane. Then because every plane touching this cone is inclined 60° , a tangent to the circular base of the cone will be the horizontal trace of such a plane, and if it be drawn as well through the horizontal trace of the perpendicular to plane 45° , it will be the horizontal trace required, i.e. of a plane perpendicular to plane 45° and inclined itself 60° .

(4) Determine the *plan* of the intersection of the two planes. One point in this plan will be at the intersection of the horizontal traces of the two planes, and a second point will be the plan of the vertex of the cone. The line drawn through these two points will be the plan of the intersection required. 'Construct' this line into the horizontal plane, draw the square face of the cube on it so 'constructed,' and determine plan of the face therefrom. Complete plan and elevation of the cube as in former problems.

2. *Plan and elevation of a prism with base a regular hexagon. Plane of one face inclined 48° . Base 68° .*

(1) Determine the plan of the rectangular face of prism as for the face of the cube first determined in the last problem.

(2) On a vertical plane at right angles to plane 68° , i.e. with its ground line taken at right angles to the horizontal trace of plane 68° , determine the vertical trace of plane 68° , and 'construct' this plane into the horizontal plane. On AB the edge common to plane of face and plane of base so 'constructed,' describe the

regular hexagon. Determine plan therefrom and complete problem as in former cases.

3. *Plan and elevation of a tetrahedron. Planes of adjacent faces θ° and ϕ° .*

(1) Determine the angle α° between the two adjacent faces of the solid.

(2) Draw the traces of plane θ° .

(3) Determine a second plane inclined ϕ° and making an angle of α° with plane θ° , Chapter I., Problem 26. The intersection of these two planes θ° and ϕ° is the indefinite edge of the solid. 'Construct' plane θ° or ϕ° , and on the intersection so 'constructed' mark off the length of the given edge of the tetrahedron, describe the face upon it, and proceed as in former problems.

VII.

SECTIONS BY OBLIQUE PLANES.

1. *To find the section of a prism standing on one of its bases when cut by an oblique plane which is given by its traces.*

Assume auxiliary vertical planes with their horizontal traces parallel to the horizontal trace of the given cutting plane, and containing the edges of the prism. The points in which these edges are cut by the plane of section will be in the intersection of this plane with the assumed vertical planes. The heights at which they are cut will be shown by the heights at which the vertical traces of the sectional plane and assumed planes cross. These heights can readily be transferred to the proper edges in elevation, and the section required completed by joining adjacent edges.

To find the true shape of the section.

This can be done by 'constructing' the plane of section, and with it the section itself, into the horizontal or vertical plane of projection. E. g. to 'construct' point $p\ p'$ into the horizontal plane. Through p draw pm perpendicular to the horizontal trace and meeting it in m . Measure the hypotenuse of the right-angled triangle

which has pm for its base, and the height of P for its perpendicular, from m along mp produced. This will give point P 'constructed.' If the compasses be held from m to P and then turned down to P 'constructed,' the student will be able to follow the path of the point P and line Pm while being 'constructed.' If the other points of the section be similarly 'constructed' and joined, we have the section in true shape.

To draw the development of the frustum of the solid, i. e. to determine the plane surface which if wrapped round the solid would exactly cover it.

The base and the true shape of the section being known, it remains to find the true form of the faces. This can be ascertained by setting along a straight line the true distances of the edges, in proper succession, erecting perpendiculars from these points for the edges themselves, and measuring the height of each edge on the line representing it, and then joining adjacent points.

To draw a projection of the frustum on its plane of section.

From p' and the other points of the elevation of the section, draw perpendiculars to the vertical trace, and mark on them from the vertical trace the actual distances of the points P , &c. from that trace, i. e. the hypotenuses of the right-angled triangles of which perpendiculars from p' , &c. on vertical trace, and from p , &c. on xy , are the sides. Join the consecutive points for the section.

For the rest of the frustum, i. e. for the points A , B , &c. of the base,

(1) Determine perpendiculars from the points of the base A, B, C , &c. to the plane of section.

(2) Find intersection of these perpendiculars with that plane, as oo' , the point where the perpendicular through A intersects the plane.

(3) Then proceed with these points of intersection as with the points of the section of the solid.

To save work, when one point as o'' in the required projection of the base is found, join o'', p'' , and draw parallels to this line through the other points q'', r'' , &c. Perpendiculars to vertical trace through the elevations b', c' , &c. will intersect these parallels in the points b'', c'' , &c. required, which may be joined consecutively to complete the projection sought.

2. *Take a cylinder resting on its base and cut it by an oblique plane. Determine the true form of the section and the development of the frustum of the solid.*

Divide the circumference into any number of equal parts, and work as if the points so obtained were the plans of the edges of a prism. The section will be an ellipse.

Converse of this Problem.

Given ab the major axis of the ellipse forming the section of a right cylinder by the horizontal plane when the axis of the solid is inclined θ degrees; to determine base and plan of frustum of solid.

To determine the minor axis of the ellipse, which it may be observed is equal to the diameter of the required base.

The vertical plane through ab contains the axis of the cylinder, and is at right angles to its base. Revolve this plane into the horizontal. Bisect ab in o , and draw op' making the angle bop' equal θ degrees. Through p' draw $l'p'm'$ at right angles to op' , and meeting parallels to that line through a and b in l' and m' . Then $l'm'$ gives the length of the required minor axis cd . Draw the curve.

Through p' a perpendicular to ab produced meets that line in p which is the centre of the plan of the base of the cylinder, which plan is likewise an ellipse having its major axis nq equal and parallel to the minor of the former; its own minor lm being determined by perpendicular on ab drawn from l' and m' . Tangents to these two curves complete the plan of the frustum of the cylinder.

3. *To determine the section of a pyramid when standing on its base and cut by an oblique plane given by its traces.*

Refer to Chapter I. Problems 12 and 14, for constructions which the student should comprehend before working this problem.

Take a pyramid with base a regular hexagon. Assume auxiliary vertical planes containing the slant edges of the pyramid. As these planes contain, and the given plane of section cuts the slant edges, the required points of section in these edges must be in the intersections of the oblique plane with the auxiliary vertical planes. Determine these intersections. The points at which the elevations of the intersections cross the elevations of the slant edges of pyramid will be the elevations of the points of section required. From them determine their plans and join adjacent edges.

The work may be shortened by finding the point in which the axis is cut by the given plane of section. As this point is common to all the intersections of planes to be found, it may be used, when once found, for all planes as a point in the elevation of their intersections. A second point in the required elevation may be readily determined by working from the junction of either the horizontal traces or vertical traces of the planes under consideration according to convenience.

To determine the true shape of the section. 'Construct' the plane of section.

To draw a projection of the solid on its plane of section.

(1) The projection of the section of the pyramid may be found similarly to the section of the prism in Problem 1.

(2) The projection of the vertex on the plane of section as the projection of any one point of the base of the prism on the plane of section in the same problem.

(3) Then lines through this vertex v'' and the points of section just determined, will be the indefinite projections of the slant edges of the pyramid. The points of the base may be fixed by perpendiculars from their elevations to the vertical trace to intersect these indefinite projections of the edges, and when found may be joined consecutively to complete the required projection of the frustum.

The *development* of the solid may be found by describing a circle with radius equal to a slant edge of the pyramid, setting along the circumference the edges of the base as chords, and joining these points to the centre of the circle. The points of section may be marked by dividing the edges

in the same proportion in which they are divided by the plane of section as indicated either in the plan or elevation.

4. *Take a pyramid standing on its base with any number of unequal sides in its base, and cut it as in the last problem by an oblique plane. Determine the true shape of section, and development of solid.*

5. *Determine the section of a cone by a given oblique plane, and the true shape of the section.*

Divide the circle which is the plan of the base into any equal number of parts and join each point in the division to the vertex. Consider these lines as the plans of the edges of a pyramid and work accordingly. The section will be an ellipse.

Converse of this Problem.

Given ab the major axis of the ellipse which forms the oblique section of a right cone by the horizontal plane, given also vv' the vertex; to draw the plan of the frustum. Fig. 28.

Since the plan v in ab produced is given, it will be obvious that when the ellipse is obtained, the required plan will be completed by drawing tangents to the ellipse through point v . Hence the problem consists chiefly in finding cd , the minor axis of the curve. This line is the horizontal trace of a circular section of the cone made through point o' , the bisection of ab , and forms a chord of that circle.

This circle being perpendicular to the axis VP of the cone, is necessarily inclined to the horizontal plane at an angle which is the complement of the angle of inclination of that axis. This circle must therefore be constructed and its chord exhibited.

VIII.

MISCELLANEOUS PROBLEMS.

1. *Plan and elevation of a cube of 3 inches edge, when three of its corners are 1, 1.7 and 2 inches respectively above the horizontal plane.*

Refer to Chapter 1. Problem 21.

Take 1 inch to equal 10 units.

(1) Draw the square $ABCD$, and figure A as 1 inch or 10 units, B as 17 units, and C as 20 units, that is, the respective heights to which these points are to be lifted.

(2) To find a 'horizontal,' in order that the square may be folded about it into the required position.

If A and C , which are to be at heights of 10 and 20 units, be joined, there will be a point in the line AC 17 units high, that is, at the same level as point B . Find this point M by dividing the line in due proportion, and the line drawn through B and M will be the horizontal required at level 17 units or 1.7 inches above the horizontal plane.

(3) A vertical plane at right angles to plane of face $ABCD$ may now be assumed by taking a line at right angles to BM for the intersection, xy , of this vertical plane

and the horizontal plane at level 17 units. The vertical trace of plane of face $ABCD$ may be easily determined because a' and c' will by the position of the assumed vertical plane be in the vertical trace and their heights are known.

(4) The square $ABCD$ is the lowest face of the cube 'constructed' in a horizontal plane at level 17 units, about the horizontal BM . The rest of the work therefore is similar to that in former problems.

2. *Plan and elevation of a tetrahedron, edge 3 inches, when the three corners of its lowest face are .8, 1.5 and 2.2 inches above the horizontal plane.*

3. *Plan and elevation of a cube of 3 inches edge, when the indefinite plans of three adjacent edges meeting in point A are given.*

Compare Chapter 1. Problem 27.

(1) Draw the three given lines as ab , ac , and ad forming 3 obtuse angles at a . Because the projections of a perpendicular to a plane are at right angles to its traces, a line drawn at right angles to any one of these edges as to ab , will be a horizontal in the plane containing the other two. Draw such a line to cut ac in p and ad in s . Then the triangle pas is the plan of a right angle which may be 'constructed' about the horizontal ps by bisecting ps and describing a semicircle on it. The angle in a semicircle being a right angle, point A when 'constructed' will be in the intersection of the semicircle with ab . If p and A , and A and s be joined, the edge of the cube, 3 inches, can be measured from A on these lines Ap and As , or the same produced, and the square can be completed.

(2) The plan and elevation of the whole may be finished by taking a vertical plane at right angles to the horizontal ps and working from the square 'constructed.'

4. *Given an oblique plane by its traces, neither being perpendicular to xy . Determine the plan and elevation of a tetrahedron resting upon it.*

Determine the inclination of the plane, Chapter 1. Problem 5. Take a new ground line at right angles to the horizontal trace, set up the vertical trace in this new vertical plane, complete elevation and plan therein, and thence derive the elevation upon the first or given vertical plane.

5. *An oblique pyramid stands on its base, which is a regular hexagon $ABC...F$ of 1.5 inches side. V is the vertex, the face VAB is inclined at 75° to the base, VBC at 65° , VCD at 60° . Draw the plan of the solid.* Science Examination, 1870.

(1) Draw the hexagon and determine the trace of the plane of face 75° on a vertical plane at right angles to it.

(2) Determine the traces of plane 65° on a vertical plane at right angles to it, and transfer them when found to the first vertical plane, determined for plane 75° .

(3) Find the intersection of these two planes which will contain V and one edge VB of the solid.

(4) Determine similarly the intersection of planes 65° and 60° , taking care that all three planes are referred to the same horizontal and vertical planes of projection. The intersection of planes 65° and 60° contains the edge VC . The

intersection of the *three* planes gives the vertex *V*—refer to Chapter I. Problem 12, Cor. 1.

The rest of the work is simple.

6. *Given the plan of a sphere, required to determine the plan of a cube inscribed in it, a the plan of one of its corners, and the direction ab or plan of one edge being also given.*

(1) To find the true length of the edge of the inscribed cube. On the diameter of the circle which is the plan of the sphere, mark off one-third of its length from one extremity, and from the point so found erect a perpendicular to meet the circumference. Join the extremities of the diameter to this point in the circumference. The shorter line is the edge of the inscribed cube, whilst the longer would be the edge of an inscribed tetrahedron. Or, by computation, if *d* equals the diameter of the sphere in inches, and *e* the required edge of the cube, then $\sqrt{3} : \sqrt{1} :: d : e$.

(2) Make an elevation of *AB* on *ab* used as a line of level at the height of the centre of the sphere, and determine *b*. To do this, describe a circle on the part of *ab* produced which cuts the circle forming the plan of the sphere. Erect a perpendicular to *ab* from point *a* to meet the circumference. This gives *a'*. From *a'* measure *AB* as a chord. This gives *b'*, and from *b'* a perpendicular to *ab*—which being used as a line of level corresponds for this work to a ground line—determines *b*.

(3) A line drawn from *b* through the centre *o* of the sphere with the distance from *b* to *o* measured on it from *o* will give the plan of *g*, another point in the face *A...G*.

(4) Determine the plane by its traces containing *aa'*, and perpendicular to *ab*, *a'b'*. This will be the plane of the

face $A...G$ of the cube. To do this, through point a' draw the vertical trace at right angles to $a'b'$, and through m where this vertical trace crosses ab , the line of level, draw horizontal trace at right angles to ab .

(5) 'Construct' this plane and with it points aa' , gg' . Join AG which will be a diagonal of this face of the cube constructed. Complete the square, and therefrom determine its plan by means of the traces of its plane already found.

(6) The edge ab being already drawn, parallels to ab and of the same length through the point $a...g$ will complete the plan of the solid.

7. *A pyramid, 5 inches high, has its axis inclined 50° and one diameter AD, 4 inches long, of its hexagonal base inclined 25° . Draw plan and elevation, and determine the circumscribing sphere.*

(1) If the axis is inclined 50° , the plane of the base will be inclined 40° . Determine this plane by its traces, and draw plan and elevation of the solid resting on it. Refer to Chapter IV. Problem 1.

(2) To determine the sphere. 'Construct' the plane containing the vertex and a diameter of the base of the pyramid. The horizontal traces of the two slant edges of the pyramid springing from this diameter to the vertex will be in the horizontal trace of plane required. Determine the centre of the triangle formed by this diameter and the slant edges of solid thus constructed, and describe circle therefrom passing through the three corners of the triangle, and you have the centre of the required sphere and its radius. If this point be S , its projections s and s' can be readily determined.

8. *To determine the centre and radius of a sphere, on the surface of which four given points are situated.*

Care must be taken not to assume these points so that any three are in the same straight line : and the fourth point cannot be situated in the same plane with the other three, unless the four are in the same great circle of the sphere.

If the line joining any two points be bisected by a plane perpendicular to it, since every point in that plane is equidistant from the extremities of the line which it bisects, the plane must contain the centre of the sphere. For the same reason if two other planes be assumed, similarly bisecting any other two of the lines joining the given points, three planes will have been determined, each containing the required centre of the sphere. It will therefore be in their common intersection, that is, in the point in which the three planes intersect.

The length of the radius will be found by means of the right-angled triangle. If the distance from the plan of the centre to that of any point on the surface be taken as the base of a right-angled triangle, and the difference of level between the centre and this point as the perpendicular, the hypotenuse will be the radius required.

9. *Draw the cylinder which would envelop a sphere of 1·5 inches radius and 1 inch above the ground. Let its axis be inclined 50° , and make in plan an angle of 30° with the ground line. Show circle of contact and horizontal trace of cylinder.*

(1) Draw sphere and plan of axis of cylinder according to conditions.

(2) Use the plan of the axis as the ground line for a new elevation, in which draw a second elevation of the sphere. A line through the centre of sphere meeting the new ground line at the given inclination will be the elevation of the axis, and parallels to it, tangential to the circle which represents the sphere, will give the major axis of the ellipse in which the cylinder cuts the horizontal plane. The minor axis of the curve will be marked off by parallels to the axis of the cylinder in plan, tangential to the circle which is the plan of the sphere. From the second or auxiliary elevation, the elevation on the original vertical plane can be easily determined.

(3) The circle of contact is shown, in the auxiliary plane of elevation, as a straight line, and from this can readily be determined in plan, and from thence on the original vertical plane.

10. *A cone, with vertex 5 inches high, axis inclined 50° and in plan making an angle of 30° with the ground line, envelops a sphere of 1 inch radius and 1 inch high. Determine horizontal trace of cone and circle of contact.*

As in the last problem use the plan of the axis as the ground line for an auxiliary elevation. In this auxiliary plane set up the elevation of the axis by drawing a line inclined 50° with xy through the point which is the elevation of the centre of the sphere, and determine vv' the vertex of the cone 5 inches high. Tangents from v' to the circle representing the elevation of the sphere will mark off on xy the line which is the elevation of the horizontal trace of the cone. This same line is also the length of the major axis of the ellipse which is the horizontal trace of the cone.

To find the minor axis of the same ellipse, the point must be determined on the circle of contact, through which a line from the vertex to the extremity of the minor axis would pass. In the auxiliary elevation this line will be shown by joining the vertex and the centre of the major axis. The intersection of this line with the straight line representing the elevation of the circle of contact gives the elevation of the point required, through the *plan* of which point a line drawn from the plan of the vertex marks off the extremity of the minor axis sought.

But note that the straight line which in the auxiliary elevation will be the elevation of the circle of contact will not pass, as in the case of the cylinder, through the elevation of the centre of the sphere. It is the line joining the tangent points before determined.

The plan of the circle of contact can be readily determined by previous problems.

The traces on the horizontal plane would represent the shadows of spheres cast by rays of light which are parallel in the case of the cylinder, and which proceed from a fixed point in that of the cone.

The circles of contact would separate the dark and light parts of these spheres.

IX.

TANGENT PLANES TO CONES.

PROBLEM I.

To determine a tangent plane to a cone at a given point A on its convex surface.

Let V be the vertex of the cone. Join points A and V : the indefinite straight line AV is the generatrix of the conic surface. Determine the 'trace' of the line AV and that of the conic surface, the former will be a point in the latter. At this point determine a tangent to the trace of the conic surface; the plane of this tangent and AV is the tangent-plane required.

PROBLEM II.

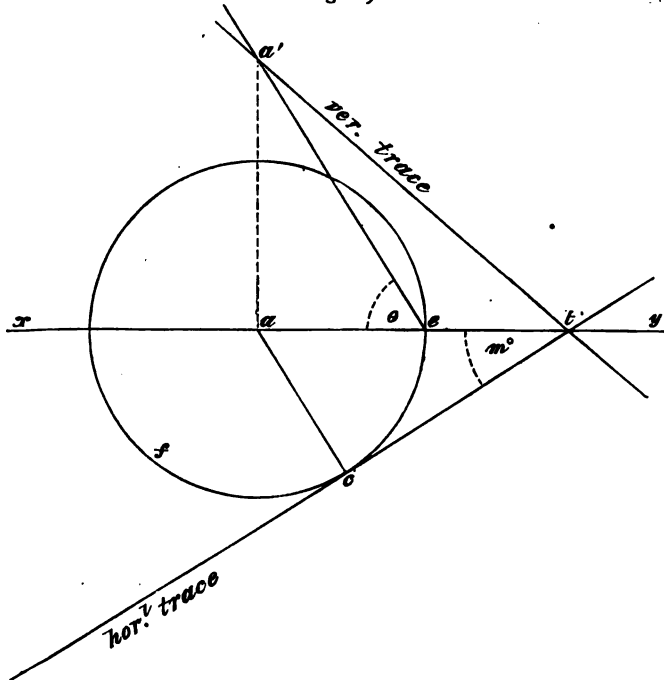
To determine a tangent-plane to a cone, which shall also pass through a point P which is external to the conic surface.

If V be the vertex of the cone: join points V and P and obtain the horizontal trace of the indefinite line VP ; through this point H draw a tangent to the horizontal trace of the conic surface; the plane of VH and this tangent is the tangent-plane required.

PROBLEM III.

To determine the traces of a plane inclined θ degrees, the horizontal trace of the plane making an angle of m degrees with xy . See Fig. 29.

Fig. 29.



Assume a point aa' in the vertical plane; make this point the vertex of a right cone generatrix inclined θ degrees. Let ac be the circular base of the cone. Draw ct a tangent to ac at c , meeting xy at t , and making the angle xtc an angle of m degrees; join $a't$, then cta' is the required plane; being a tangent-plane to the cone, which it touches in the line AC .

PROBLEM IV.

The axis of a given right conic surface is oblique to the horizontal plane. Required a tangent-plane to this surface, which shall have an inclination of θ degrees.

Determine the horizontal trace of the given cone, viz. the ellipse, see figure Case 6, Chap. XII. Spherical Triangles: likewise the circular horizontal trace *ts* of a cone having the same vertex *tv'* with the given cone, and generatrix inclined θ degrees (*A* degrees in fig.): the common tangent-plane to these two cones, which has *lt* for its horizontal trace, is the tangent-plane sought.

TANGENT PLANES TO SPHERE.

PROBLEM I.

To determine a tangent plane to a given sphere centre cc' , at a given point qq' on its surface.

If the sphere rests on the horizontal plane in point *Q*, then that plane is the tangent-plane required.

If *Q* be the highest point on the sphere, the tangent-plane is likewise horizontal.

If *Q* be situated on the horizontal great circle of the sphere the tangent-plane will be a vertical one.

Such plane may make any angle from zero to 90° with the vertical plane of projection; according to the position of point *Q* on the great circle.

For any other position of the point Q the tangent plane will be an oblique one, making of course an angle of 90° with QC its normal, and therefore inclined at an angle which is the complement of that of the inclination of the line QC . It may therefore be drawn at right angles to QC by the converse of Problem 13, Chap. I.

PROBLEM II.

To determine a tangent plane to a given sphere centre, cc' , which shall pass through a given point qq' which lies without the spherical surface.

If tt' or T be the point in which the required plane touches the sphere; then the radius CT is the normal of the plane and is perpendicular to the straight QT in it; and QTC is a right-angled triangle, having the angle QTC a right angle.

Assume any plane passing through the points Q and C . This plane cuts the sphere in a great circle, a tangent to which through Q determines the point of contact of the tangent plane.

If the triangle QTC revolve about the line QC , it will be observed that the 'locus' of the point T is a small circle of the sphere. The solutions of the problem are therefore as numerous as the points on this circumference, which is the base of a cone with vertex in Q enveloping the sphere.

NOTE. That if the line QC be a horizontal one, the *plan* of this circle is a chord of that which forms the plan of the sphere.

PROBLEM III.

To determine a tangent plane to a given sphere, centre C , which shall also pass through two given points P and Q , which are without the spherical surface.

The problem is a possible one if the straight line which passes through P and Q does not cut the surface of the sphere.

Determine, as in Problem 2, the 'circle of contact' of a cone enveloping the sphere, and having its vertex in the point P .

Determine the circle of contact of a cone enveloping the sphere and having its vertex in the point Q .

Let these circles intersect in points R and S . Then the planes of points P, Q, R and P, Q, S are tangent planes to the sphere.

It will shorten the work if when neither P nor Q lies in the horizontal plane of C , a point V in the line PQ and in that plane be determined and made the vertex of the second cone; since the plan of the circle of contact will then be a straight line (see Note to last Problem) instead of a second ellipse.

A second solution of Problem III.

Take a plane through the centre C of the sphere and at right angles to the straight line which passes through points P and Q . Let I be the intersection of this line and plane. Then straight lines through point I touching the great circle in which this plane cuts the sphere, give points R and S : and the planes of the three points P, Q, R and P, Q, S are the tangent planes.

TANGENT PLANES TO SPHERE AND CONE,

PROBLEM IV.

A right cone, 4 inches high, has its base (the diameter of which is 3 inches) in the horizontal plane; a sphere (radius 1 inch) has its centre 3 inches high and distant 4 inches from the axis of the cone. Determine the common tangent planes to the solids.

Envelop the sphere by a cone similar to the given one, and having its base in the horizontal plane; the common tangent planes to these cones are the planes required. Their horizontal traces touch the circular bases of the cones, and may be produced to meet the xy of any assumed plane of elevation. As these planes contain the vertices of the cones, the straight line through these points will give a second point in the vertical trace of each tangent plane.

PROBLEM V.

Three spheres, A, B, and C, of 1.6, .8, and .4 inches radii respectively, lie on the horizontal plane, each sphere touching the other two. Draw plan and elevation, and determine a second tangent plane to them.

(1) To determine the three spheres in position. Draw plan and elevation of A , the largest. Draw an elevation of B tangential to that of A , and determine plan of B therefrom, taking care that the plan of B is so placed that the vertical plane containing the centres of A and B is parallel

to the vertical plane of projection. Draw two elevations of C , one tangential to that of A and another to that of B . Take the horizontal distances between the elevations of the centres of A and C , and of B and C , and from the plans of the centres of A and B , with these distances for radii respectively, describe arcs. The intersection of these arcs will determine the plan of the centre of C , the third sphere. Its elevation can then easily be obtained.

(2) To draw the tangent plane to A , B and C . Determine plan and elevation of axes of cones enveloping A and B , and A and C . Lines through the centres of the spheres give these axes. The horizontal traces of these axes will be two points in the horizontal trace of the tangent plane required, since the tangent plane to the cones must contain their vertices which are by their position in the horizontal plane. The tangent plane to the cones is the tangent plane to the spheres they envelop. The vertical trace may be determined by taking a vertical plane at right angles to the tangent plane, and drawing the vertical trace tangent to an elevation of any one of the spheres. This vertical trace can then, if required, be transferred to the original plane of elevation.

PROBLEM VI.

To determine the tangent planes to three spheres, A , B and C , of unequal sizes and of unequal heights.

(1) Determine two cones enveloping A and B , and A and C respectively. Tangents to the projections of the spheres will determine the projections of the cones. The line passing through the vertices of the two cones will be

in the required plane, and its horizontal trace in the horizontal trace of that plane, because the tangent planes must touch the cones and so contain their vertices.

(2) Determine the two circles of contact of the cones with sphere A . The plans of these are ellipses. The points in which the ellipses—see Note to Problem 2, ‘tangent planes to spheres,’ by attention to which one of these ellipses becomes a straight line—thus determined intersect will be tangent points in the required planes, and if these points be joined to points taken in the line passing through the vertices of the cones, the horizontal traces of such lines will be in the horizontal traces of the required tangent planes.

PROBLEM VII.

To determine a cone, with axis vb , $v'b'$ and vertex vv' given, which shall touch a given sphere, centre cc' .

(1) Determine the horizontal trace of the plane containing given axis of cone and centre of sphere, and make an elevation of this plane on a vertical plane assumed at right angles to it. Observe that this plane cuts the given sphere in a great circle and given cone in a generatrix VM , which is a tangent to that circle in T or t' the point of contact of the cone and sphere.

(2) Turn this plane into the horizontal plane about its trace, ‘constructing’ at the same time points V and B , and the circle in which the plane cuts the sphere. From V draw tangent VM to touch the circle turned down, at point T , and determine therefrom t , the plan of the point in which the cone touches the sphere.

(3) If V and B be joined, we have the axis 'constructed,' and a line from B at right angles to VB to meet VM , the tangent to the circle at M , completes the triangle VBM , which is the 'constructed' half section of the cone made by the plane containing axis and centre of sphere. The line BM is, therefore, the radius of the base of the cone, and may be measured from b at right angles to vb for the semi-major axis of the ellipse, which is the plan of the base, this axis being horizontal.

(4) To determine the minor axis of the ellipse, make a third elevation of the axis of cone, using the plan of axis of cone for a ground line. In this elevation a section of the cone in true shape will be shown by drawing a line $b's'$, equal to BM , from b' at right angles to $v'b'$, and completing the triangle $v'b's'$. The side $b's'$ is an elevation of the minor axis of the ellipse required. The plan bs can then be determined, and the plan of the cone completed.

Obs. The two solids have a common tangent plane at their point of contact T , which touches the cone in the line TV . This plane may therefore be drawn at right angles to CT .

TANGENT PLANES TO CYLINDER.

PROBLEM I.

To determine a tangent plane to a given cylinder at a given point qq' on its surface.

Determine a generatrix of the cylinder at the given point Q , by drawing through q and q' lines parallel to the respective projections of the axis of the solid.

Determine the trace of this generatrix and that of the cylindrical surface. The former is a point in the latter. The straight line through the former, tangential to the latter, is the *trace* of the required tangent plane.

Obs. (1) If the axis of the given right cylinder be vertical, its horizontal trace is a circle, and its tangent-plane may make an angle varying between zero and 90° with an assumed vertical plane.

(2) If the axis be horizontal, the angle of inclination of the tangent plane varies between zero and 90° , according to the position of the point Q .

(3) If the axis be oblique, the horizontal trace of the surface is an ellipse and the inclination of the tangent-plane varies, according to the position of point Q , between 90° and the angle of inclination of the axis.

PROBLEM II.

To determine a tangent plane to a given cylinder, passing through a given point qq' which is without the surface of the solid.

Through the given point Q draw a straight line parallel to the axis of the cylinder, and determine its trace.

Determine the corresponding trace of the cylindrical surface.

A tangent to the latter drawn through the trace of the line is the trace of the required tangent plane.

PROBLEM III.

To determine a tangent plane to a given cylinder, which shall have a given inclination θ .

For reasons before indicated, the angle of inclination θ of the tangent-plane cannot be less than the angle of inclination of the axis of the given cylinder.

Determine a plane inclined θ° containing the axis of the cylinder: the plane parallel to this and touching the cylindrical surface is the required tangent plane.

PROBLEM IV.

To determine a cylinder which shall have a given straight line for its axis and shall touch a given cylinder.

It is obvious that the two cylindrical surfaces will have a common tangent plane at their point of contact. The

problem consists therefore in the determining this point; which will lie in a straight line at right angles to both axes.

Let AB be the axis of the given cylinder, and CD that of the required one: AB not lying in the same plane with CD . Determine the line PQ at right angles to each of these lines by Problem 23, Chap. I., meeting AB in point P and CD in Q .

Let PQ pierce the surface of the given cylinder in point S , then S is the point of contact required, and QS is the radius of the base of the required cylinder.

PROBLEM V.

Given two points, B and C, at equal distances from a given plane, as the foci of the elliptic section of a right cylinder which touches the plane: determine the cylinder and its line of contact with the plane.

The ellipse is perpendicular to the given plane, and touches it in the extremity Q of the minor axis of the curve. This point may be determined by a perpendicular to the straight line BC and the plane, through O the point of bisection of BC ; Q being the intersection of this perpendicular and the plane.

Complete the ellipse by making its major axis equal to $BQ + CQ$.

The position of a circular section of the cylinder, perpendicular to the plane, with radius equal to OQ can be

determined, with one extremity of a diameter coinciding with an extremity of the major axis of the ellipse. See the construction given in Chap. VII. converse of 2.

The straight line, through point Q , and the point in which the circle touches the plane, is the required line of contact of the cylinder and its given tangent plane.

Work the problem :

- (1) When the given plane is horizontal,
- (2) is inclined.

X.

INTERSECTIONS OF SOLIDS WITH PLANE SURFACES.

NOTE. Developments should be drawn to all the problems of this Chapter, for which refer to Chapter VII.

PROBLEM I.

Two right prisms, with square bases of 2.5 inches side, intersect. One is placed with its edges horizontal and parallel to the vertical plane, and with its lowest face inclined 30° and its lowest edge 1 inch above the horizontal plane. The other stands on its base with one face inclined 25° to the vertical plane and with its axis passing through the highest edge of the horizontal prism. Other dimensions at pleasure. Draw plan and elevation.

Commence by drawing an auxiliary elevation of the horizontal prism, so arranged, that the elevations of both of its bases coincide: that is, draw the square and a ground line inclined 30° to one side and 1 inch below the lowest corner or angle of square. From this determine plan.

The plan of the vertical prism can then be easily placed in position, and will be a square with one side inclined 25° to the ground line of the required elevation.

Then proceed with the required elevation. The points in which the plans of the edges of the horizontal prism meet the square, which is the plan of the vertical prism, will be the plans of the points of intersection between those edges and the vertical faces; and their elevations can readily be found by perpendiculars from them to the ground line to meet the elevations of the same edges.

The points in which the vertical edges of the one prism intersect the faces of the other will not be evident in plan from the vertical position of those edges, but they can be marked off in elevation by transferring their heights, which can be readily seen in the first or auxiliary elevation, to the required one.

The proper points must then be joined, and this will best be ascertained by inspection from the auxiliary elevation.

PROBLEM II.

Take the same prism similarly arranged, except that the horizontal edges of the one are inclined 30° to the vertical plane, and determine plan and elevation.

PROBLEM III.

Let the horizontal prism be intersected by one with base a regular hexagon of 1.5 inches side standing on its base, and with one edge at least outside the other solid. Other arrangements at pleasure.

PROBLEM IV.

Plan and elevation of a right prism, with bases equilateral triangles of 2 inches side, piercing horizontally a 2 inch square pyramid standing on its base. No face of prism to be horizontal or vertical. All horizontal edges to enter pyramid. One side of the base of the pyramid to be inclined 30° with the vertical plane. Other dimensions and arrangements at pleasure.

(1) Make an auxiliary elevation at right angles to the horizontal edges of the prism. This elevation of the prism will be a triangle.

(2) The points where the elevations of the edges of the pyramid cross this triangle will be the elevations of the points of intersection of the edges of the pyramid with the faces of the prism. Perpendiculars to the xy from these points will be loci of their plans.

(3) To find the points in which the horizontal edges enter and leave the pyramid, draw in the auxiliary elevation lines from the vertex to the base passing through the elevations of the horizontal edges. These are really lines on the faces of the pyramid passing through the points where the horizontal edges of prism enter and leave the pyramid. Determine these lines in plan. The points where they cross the plans of the horizontal edges of the pyramid are the plans of the points required.

(4) The proper points must then be joined from inspection.

PROBLEM V.

Take a pyramid and prism of four and five sides in each base respectively, and find intersection. Let the pyramid be placed as in the last problem, and the prism with horizontal edges, and with two entirely free from the pyramid.

PROBLEM VI.

To determine the penetration of a prism standing on its base with a pyramid, when the base of the pyramid is inclined θ° , has no side horizontal, and is not at right angles to the vertical plane.

Draw pyramid and prism, preparatory to finding their interpenetration.

The plans of the points at which the edges of the pyramid enter the prism are shown where these edges in plan cross the plan of the prism, and can thence be determined in elevation.

The intersection of the vertical edges of the prism with the faces of the pyramid can be found, *either*, by determining the traces of the planes of those faces and finding their intersection with the vertical edges passing through them, *or*, which is more simple, by drawing lines from the plan of the vertex of the pyramid through the plans of the edges of the prism to the base of the pyramid, and showing these lines, which are really lines on the surfaces of the faces of the pyramid, in elevation. The intersections of the elevations of these lines, with the ele-

vations of the edges of the prism through which they were drawn in plan, give the elevations of the points required.

The proper points must then be joined from inspection, so as to give the whole interpenetration of the two solids.

PROBLEM VII.

To determine the interpenetration of two or more solids with plane surfaces, by means of parallel sections.

Take, for example, two irregular hexagonal pyramids, of assumed but different heights, resting on the horizontal plane and intersecting one another.

Determine their plans only. The faces which intersect can be seen from inspection.

Then, to find the intersection of any two such intersecting faces, draw horizontal lines in each face at the same level.

The points at which these lines *at the same level* cross one another will be points in the actual intersection of the planes of these faces.

Any two such points being determined, the line of intersection can be drawn through them until it meets an edge bounding one of the faces under consideration.

By continuing this method with all the other intersecting faces, the whole interpenetration will be determined.

XI.

INTERSECTIONS OF SOLIDS WITH CURVED SURFACES.

PROBLEM I.

Two cylinders, one horizontal with base 2 inches in diameter, the other vertical with base 2.5 inches in diameter, intersect. The axis of the horizontal cylinder is 2 inches above the ground, passes through the axis of the other, and is inclined 40° to the vertical plane. Determine plan and elevation.

Arrange the cylinders in plan and elevation preparatory to finding their intersection.

Determine horizontal lines on the surface of the horizontal cylinder in plan and elevation. To do this, fold the base in plan about its horizontal diameter until it is parallel with the horizontal plane, and divide the circle into equal parts. Lines drawn through these points parallel to the plan of the axis will be the plans of the lines required. Determine these lines in elevation by folding the base in elevation about its vertical diameter until it is parallel with the vertical plane. Divide the circle into the same number of equal parts as in the plan,

and through these points draw lines parallel to the elevation of the axis. These will be the lines on the surface in elevation. Care should be taken to begin the division of the circle in plan and in elevation at the same, say the highest point, and to mark the plans and elevations of the lines correctly for future use.

The plan of the intersection required will coincide with the plan of the base of the vertical cylinder, the points at which the horizontal lines enter the vertical cylinder being shown in plan where the lines cross the circular base.

The elevations of these points will be determined by perpendiculars from their plans to the ground line, to meet the elevations of the lines in which they must be situated. The elevation of the whole intersection will be completed by joining the elevations of these points.

PROBLEM II.

Find the intersection of two cylinders similarly arranged to those in the last problem, except that the axis of the horizontal one is .5 inch from that of the vertical cylinder and its base is 2.5 inches in diameter.

The difference between this problem and the last is, that there are not two separate curves, one in which the horizontal cylinder pierces the vertical and another in which it leaves it, but that the intersection forms one continuous curve.

The work is exactly similar.

PROBLEM III.

Determine the intersection of a vertical cylinder with base 2.5 inches, and a cone with base 2.5 inches, axis horizontal, passing through that of the cylinder, and inclined 45° to the vertical plane.

The base of the cone must be folded parallel first to the horizontal and then to the vertical plane, so as to determine lines on the convex surface, as in the preceding problem on the surface of the cylinder. Care must be taken to transfer the points by which the circle of the base when so folded is divided on to the ellipse which is its elevation, and to the line which is its plan, before joining them with the vertex. The rest of the construction corresponds to that in the preceding problems.

PROBLEM IV.

Arrange an example of cone and cylinder, with the axis of the cone free from that of the cylinder, so that the curve of intersection may be a continuous one.

PROBLEM V.

Arrange an example with cone vertical and resting on its base, and a horizontal cylinder penetrating the cone. Let the axis of the cylinder be free from that of the cone and inclined to the vertical plane, say, 45° . Let part of the cylinder be outside the cone.

Determine horizontal lines on the surface of the cylinder in plan and elevation as in preceding problems. Then determine sections of the cone by horizontal planes at the various heights at which these lines on the cylinder are.

In the elevation these sections will be straight lines, being represented by those portions of the lines on the surface of the cylinder which cross the elevation of the cone.

In the plan these sections will be circles described from the plan of the axis of the cone as a centre with the elevations of the corresponding circles for diameters.

The points in plan at which the lines on the cylinder cross the circular sections of the cone on the same level, will give points in the interpenetration of the solids.

These points when determined must be joined from inspection, and thence determined in the elevation.

PROBLEM VI.

Plan and elevation of the intersection of a horizontal cylinder with a sphere under the following conditions: sphere with radius 2 inches and centre 2.5 inches high. Cylinder with axis .5 inch from, but not on the same level with the centre of the sphere, and inclined 45° to the vertical plane. Diameter of cylinder 2 inches.

The construction is similar to that in the preceding problem, the assumed horizontal planes cutting the sphere in circles as in the case of the cone.

PROBLEM VII.

Arrange a cone placed vertically, and a sphere, and find intersection. Let centre of sphere and axis of cone be free from one another.

PROBLEM VIII.

To determine the intersection of two cylinders, the axes of which are not parallel to either plane of projection.

(1) Draw two lines making any angle, say 60° , with each other. Assume an ellipse having its major axis on one of them as the horizontal trace of one cylinder. Complete the plan of this cylinder.

(2) *To determine its elevation.* Make a vertical section on the major axis to obtain the portion of the axis of the cylinder. This is done by describing a semicircle on the major axis, and setting off from one extremity as a chord the length of the minor. A line at right angles to this through the centre of the ellipse is the axis of the cylinder.

(3) Arrange a second cylinder similarly on the second line of the angle 60° , so that its convex surface may penetrate that of the former.

(4) *To find their intersection.* Assume a point in plan and elevation, and from it draw two lines parallel to the axes of the cylinders. Determine the horizontal traces of these lines, and join them. The line thus drawn will be the horizontal trace of a plane containing these two lines.

(5) Points in the intersection of the cylinders must then be found by assuming auxiliary planes, cutting the bases of the cylinders and parallel to the plane before found, that is, with their horizontal traces parallel to the horizontal trace of this plane. As each of these cutting planes will contain lines on the surfaces of the cylinders springing from the points where it crosses their bases, and as these lines are all in the same plane, the points in which they actually cut one another will be points in the intersection required.

Any number of points can be found, by assuming a corresponding number of auxiliary cutting planes. The projections of these points must then be joined by hand to get the required curve of intersection.

PROBLEM IX.

To determine the intersection of two cones with oblique axes.

Assume two intersecting cones by the converse of Problem 5, Chapter VII.

Planes assumed passing through the vertices of the two cones and cutting their surfaces, will do so in straight lines, the intersections of which lines will give points in the intersection of the two cones.

Draw a line through the vertices of the two cones and find its horizontal trace. The horizontal traces of the cutting planes must pass through this point and cross the two bases. The rest of the work will be similar to that of the last problem.

XII.

SPHERICAL TRIANGLES.

CASE I.

Given the three sides a , b and c ; to determine the three angles A , B and C .

Let the three sides be developed on the horizontal plane of one of them, viz., b .

Take a plane of elevation xy at right angles to Ol , the common edge of the sides c and b ; and let it meet the second edge of side c in point P , and cut Ol in e and Om in f .

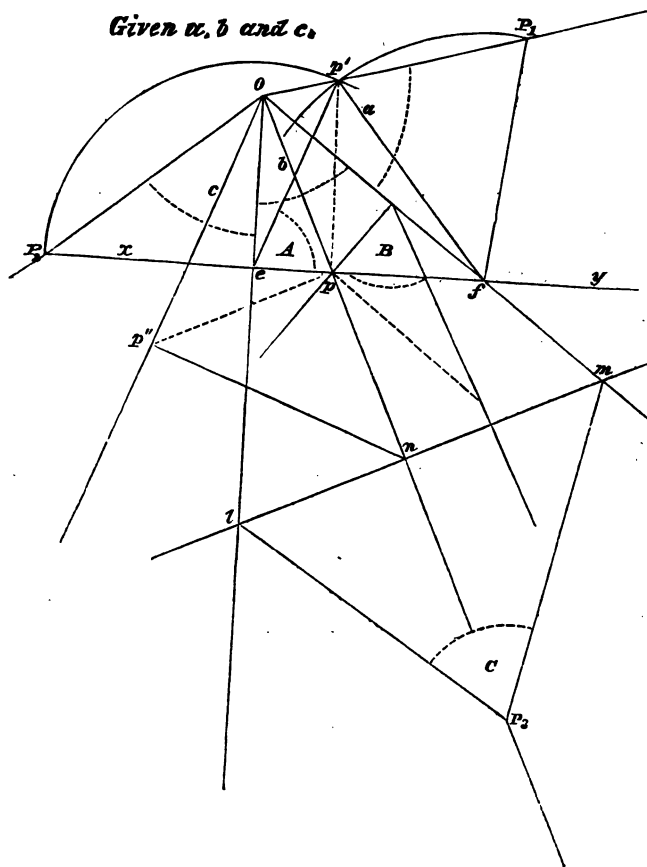
Make OP_1 on the second edge of a equal to OP ; since OP_1 and OP are really the same line, viz., the common edge of sides c and a .

Then eP , ef , fP , are the sections of the three sides by the plane xy .

Construct this section epf , in this plane; taking the centre e and radius eP , and centre f and radius fP , and describing arcs intersecting in p : join $p'e$ and $p'f$. Then p , the plan of P , is determined in xy from p' , and Op is the plan of the edge OP on the plane of b . Angle $p'ef$ is the angle A . The plane through p at right angles to Om gives the angle B . And the third angle C is determined by a plane through P , in space, at right angles to the third edge OP , in space, by Problem 15, Chap. I. lm is the horizontal trace of this plane, which is that of the profile angle of C .

CASE I.

Given a , b and c .



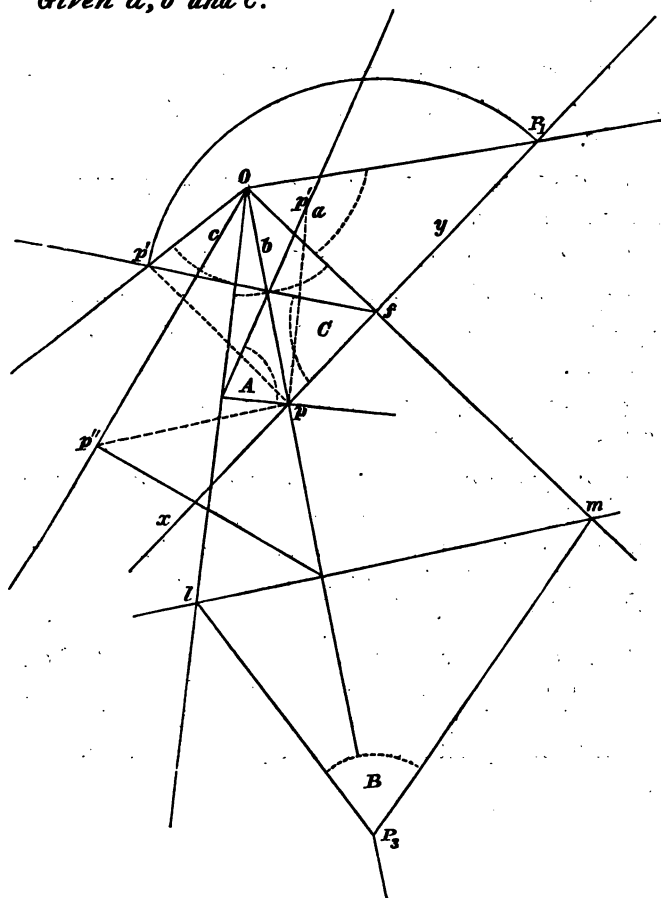
CASE II.

Given the sides a , b , and the angle C : to determine A , B and c .

Let the side a be developed on the horizontal plane of b : and since the angle C between these sides is known, a plane xy at right angles to their intersection Om exhibits this angle $pf'p'$; fp' being the vertical trace of the side a , the length fp' being made equal to fP , p in xy determined from p' , gives Op , which produced is the indefinite plan of the third edge OP on the plane of b . A plane through p at right angles to the edge Ol gives the angle A , and enables us to 'construct' the third side c about Ol its horizontal trace. A plane through P in space, and at right angles to the edge OP in space, and having lm for its horizontal trace, will determine the profile angle of B .

CASE II.

Given a, b and C .



CASE III.

Given the angles B and C and side a, that is, two angles, and their adjacent side: to determine the angle A and the sides b and c.

On the horizontal plane take the side a , of which Ol , Om are the edges.

Take the plane xy at right angles to Ol .

The vertical trace of the side c on this plane forms the angle B with xy , which angle being known this trace can be drawn.

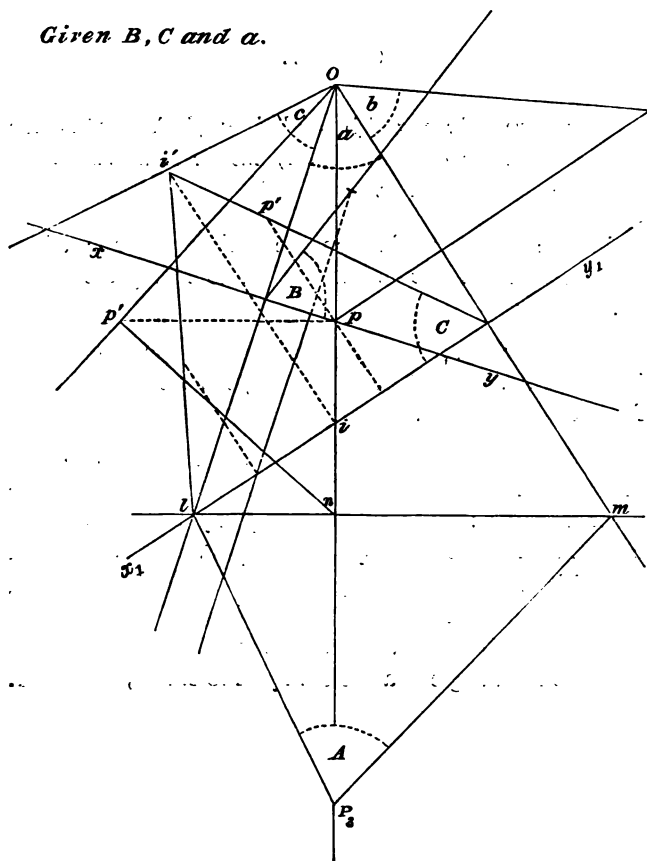
On a plane x_1y_1 , at right angles to Om , the vertical trace of side b forming the known angle C with x_1y_1 can be drawn.

By the aid of the elevation on xy determine the vertical trace of side c on plane x_1y_1 ; the two vertical traces on the latter plane intersect in a point i' , whence i is found in x_1y_1 and ii' is a point in the third edge OI , of which draw the indefinite *plan* Oi .

The faces b and c can now be readily 'constructed' on the plane of a by rotation on Om and Ol respectively. Angle A is found by Problem 15, Chapter I. as before, by a plane through point P or I and perpendicular to OI .

CASE III.

Given B, C and α .



CASE IV.

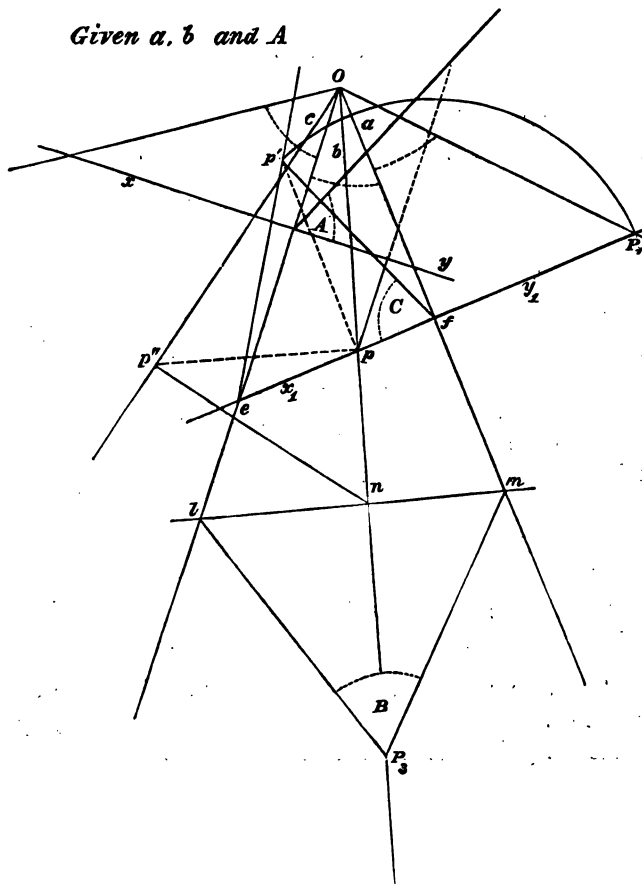
Given two sides a and b , and the angle A (that is two sides, and an angle opposite to one of them): to determine the angles B and C , and side c .

Assume the given side b on the horizontal plane, edges Ol and Om and side a 'constructed' into that plane about Om .

On plane xy at right angles to Ol make the given angle A with xy , the second line of this angle is the vertical trace of side c on this plane.

On a second plane x_1y_1 assumed at right angles to Om determine the vertical trace cp' of the side c by the aid of the former elevation: cut this trace in p' by the circular arc described on plane x_1y_1 by point P_1 on the third edge: join $p'f$, it is the vertical trace of side a ; obtain p in x_1y_1 , then pp' is a second point determined in the third edge OP of the spherical triangle. Draw the indefinite plan Op and obtain the third angle B as before by Problem 15, Chapter I,

Given a, b and A



CASE V.

Given angles B and C and a side b opposite to one of them: to determine sides a and c, and the third angle A.

Let the side b be drawn on the horizontal plane, and revolved about its edge Om until it makes the angle C with that plane.

The indefinite line fp' will then be the vertical trace of side b on the plane xy of the profile angle of C ; which is, of course, perpendicular to Om : and fp' on this trace, taken equal to fP , gives the elevation p' of a point P in the second edge of side b . p in xy , obtained from p' , determines the point pp' in this second edge: and Op is the *plan* of that edge.

Determine a plane to contain the line OP , in space, just found, and inclined at the known angle of B , by Problem 9, Chapter I: Ol , the horizontal trace of this plane, is the third edge of the required spherical triangle.

The side c can now be 'constructed' on the plane of a about Ol . The angle A may be determined as before, by a plane at right angles to OP through point P .

CASE VI.

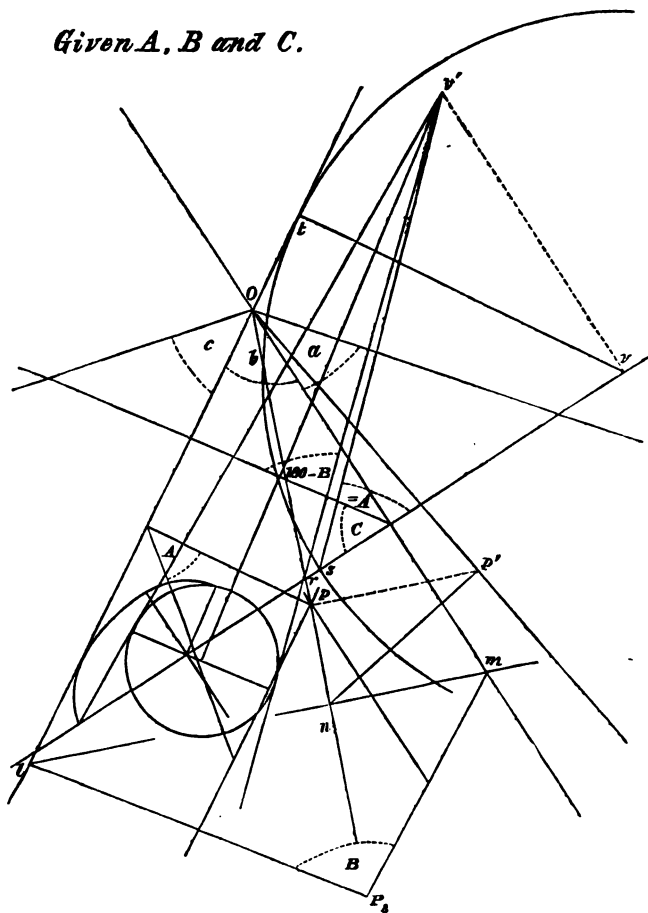
Given the three angles A, B and C: to determine the three sides a, b and c.

If the dihedral angle C has one face in the horizontal plane, the traces of the other face can be readily drawn; and of these the horizontal trace is an indefinite edge of the required spherical triangle.

A third plane making the angle of A with the horizontal, and of $180 - B$ with the inclined plane, determined by Problem 26, Chap. I., will complete the required spherical triangle. The intersections Ol , OP of this third plane with the planes of the angle C form the second and third edges of the spherical triangle. The face b will thus be determined in the horizontal plane; into which the remaining faces a and c can be 'constructed' about their respective edges or horizontal traces in the face b .

CASE VI.

Given A, B and C.



XIII.

ISOMETRIC PROJECTION.

IN Isometric Projection solid bodies the chief planes of which are mutually at right angles are so represented, that from one drawing, *a plan*, the principal measurements as to length, breadth, and height can be obtained.

The solid, the cube for instance, is to be so placed that the three planes which meet together at one corner are equally inclined to the horizontal plane, the plane of projection. The three lines which meet at that corner will then be projected so as to form three equal angles of 120° each, and will be the plans of the three edges of the cube. The plans of the opposite edges will be parallels to these, and hence it follows that in Isometric Projection all angles which are in reality right angles are projected into angles of either 120° or 60° .

Commence therefore an Isometric Problem by drawing three lines, making angles of 120° with one another, for the plans of the three lowest edges of the solid.

Then construct an Isometric scale, so as to be able to find the Isometric lengths of the required edges; measure

these, when found, on the lines drawn for their plans, and complete the problem by parallels.

To construct an Isometric scale. It is known that the relation of a line to its Isometric projection is as $\sqrt{3}$ to $\sqrt{2}$. Two lines must therefore be drawn in this relation. For this purpose take any line, say an inch, and at one end of it, erect a perpendicular of the same length, and join the ends. The square on the hypotenuse of this right-angled triangle will be equal to the sum of the squares on the two sides containing the right angle, i.e. in this case to 2 square inches, wherefore the hypotenuse itself represents $\sqrt{2}$. If from one end of this line representing $\sqrt{2}$ a perpendicular of 1 inch is erected, and this triangle completed, this hypotenuse by similar reasoning will represent $\sqrt{3}$. We now have two lines representing $\sqrt{2}$ and $\sqrt{3}$ in required relation for the scale, and if the real lengths of edges be measured along $\sqrt{3}$ and perpendiculars dropped from their ends to $\sqrt{2}$, the parts thereby intercepted will be the Isometric plans of the required edges.

I.

Isometric Projection of a brick, 9 by 4·5 by 3 inches, to half scale.

II.

Your instrument case, with the lid (1) half open, and (2) open to its full extent.

III.

Form a cross by placing on a cube of 2 inches edge four other cubes of the same dimensions.

IV.

Show the following joints in carpentry:—mortice and tenon, tongue and groove, rebate.

V.

To determine the Isometric Projection of a given circle.

(1) Describe the circle and circumscribe it with a square.

(2) Join the diagonals of this square. Join also the points where these diagonals cut the circle, thus inscribing a square in the circle.

(3) Determine the Isometric Projection of this plane figure, i.e. of these two squares.

(4) Then the required Isometric Projection of the given circle will be the ellipse drawn passing through the centres of the projections of the four sides of the larger or circumscribed square—and will have the Isometric projections of the diagonals of the smaller or inscribed square for its major and minor axes.

NOTE. That the semi-axes of the ellipse and the Isometric-radius (as it is termed, i.e. the line from the centre to a tangent-point of the curve) are in the proportion of $\sqrt{3} : \sqrt{1} : \sqrt{2}$ to one another.

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